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LOCAL HARNACK ESTIMATE FOR YAMABE FLOW ON LOCALLY CONFORMALLY FLAT MANIFOLDS*

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Abstract. In this paper, we first prove the local derivative estimate of curvature under Yamabe flow, and then by using it obtain the local Harnack estimate of Yamabe flow on locally conformally flat manifolds, under the condition $-m(t)g_{ab} \leq R_{ab} \leq Mg_{ab}$, where $0 \leq m(t) \leq M$ and $m'(t) \geq (4n+1)m(t)M$, on $t \in [0, r^2]$. As a corollary, we get a sharp derivative estimate of scalar curvature in some directions.

Key words. Yamabe flow, Local Harnack estimate.

AMS subject classifications. 53C21,53C44

1. Introduction. The Harnack estimate of geometry flow is also called Li-Yau-Hamilton inequality. R.Hamilton made important work on the Harnack estimate of Ricci flow and mean curvature flow. Recently he has found the local Harnack estimate of Ricci flow under the curvature condition $-\frac{m(t)}{2}(g_{ac}g_{bd} - g_{ad}g_{bc}) \leq R_{abcd} \leq \frac{M}{2}(g_{ac}g_{bd} - g_{ad}g_{bc})$, where $0 \leq m(t) \leq M$. It can be used to prove the second step of the proof of the Theorem7.1.1 in [1]. That was his report on the fourth ICCM in December 2007. Jie Wang got the local Harnack estimate of mean curvature flow by using the same method in [4]. The Harnack inequality of Yamabe flow was first proved by Chow on compact locally conformally flat manifolds with positive Ricci curvature in [2]. HuiLing Gu obtained the same inequality for complete locally conformally flat manifolds with nonnegative Ricci curvature in [3]. Does the local Harnack estimate also hold for the Yamabe flow ? We give an affirmative answer in this paper in the class of locally conformally flat manifolds.

Let (M^n, g_0) be a smooth complete locally conformally flat n-dimensional manifold. The Yamabe flow is given by

$$\begin{cases} \frac{\partial}{\partial t} g(x,t) = -R(x,t)g(x,t) \\ g(x,0) = g_0(x) \end{cases} \tag{1.1}$$

for $x \in M^n, t \geq 0$, and where R is the scalar curvature of g.

Let $0 \leq r \leq \theta/\sqrt{M}$, where constant θ depending only on the dimension n will be obtained in Theorem 2.1. Let $O \in M^n$, $B_r(O,t)$ is a geodesic ball centered at O, with radius r at time t. Set $d = d_t(x,O)$ is the geodesic distance function from O to x w.r.t. $g_{ij}(t)$.

Through out the paper, we denote the curvature condition

$$(\star) \begin{cases} (1) - m(t)g_{ab}(t) \leq R_{ab}(t) \leq Mg_{ab}(t) & \\ (2)\ 0 \leq m(t) \leq M & \text{on } t \in [0, r^2] \\ (3)\ m'(t) \geq (4n+1)mM & \text{on } t \in [0, r^2] \end{cases} .$$

Our main result is the following theorem:

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