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A SIMPLE APPROACH TO THE STRUCTURE THEOREM FOR NEFVALUE MORPHISMS*

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Abstract. Let L be an ample line bundle on a smooth complex projective variety X of dimension n, let τ be the nefvalue of (X, L), and let $\phi : X \to W$ be the nefvalue morphism of (X, L). A simple approach to the complete structure theorem for nefvalue morphisms with $\tau > n - 2$ is developed.

Key words. Nefvalue, nefvalue morphism.

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Introduction. In this paper varieties are always assumed to be defined over the field $\mathbb{C}$ of complex numbers.

Let X be a smooth projective variety of dimension $n \geq 1$, and let L be an ample line bundle on X. Assume that the canonical bundle K_X of X is not nef. Then, as is well known, $\tau = \min\{t \in \mathbb{R} \mid K_X + tL \text{ is nef}\}$ is a positive rational number, and τ is called the *nefvalue* of (X, L). Keep in mind that τ is the unique rational number characterized by the condition that $K_X + \tau L$ is nef but not ample. Write $\tau = u/v$ for two coprime positive integers u, v. Then the complete linear system $|m(vK_X + uL)|$ for $m \gg 0$ defines a surjective morphism $\phi : X \to W$ onto a normal projective variety W with connected fibers such that $vK_X + uL = \phi^* A$ for some ample line bundle A on W, and ϕ is called the *nefvalue morphism* of (X, L).

Assume that $\tau > n - 2$. Then the structure of nefvalue morphisms is supplied, for example, in Chapter 7 of [BS]. The purpose of this paper is to complement the above structure theorem perfectly and to offer the complete structure theorem. The precise statement of our result is as follows:

THEOREM. *Let L be an ample line bundle on a smooth projective variety X of dimension $n \geq 1$, let τ be the nefvalue of (X, L), and let $\phi : X \to W$ be the nefvalue morphism of (X, L). Assume that $\tau > n - 2$. Then one of the following holds*:

(i) *$\tau = n + 1$, $\phi(X)$ is a point, and $(X, L) = (\mathbb{P}^n, \mathcal{O}_{\mathbb{P}^n}(1))$.*

For $n \geq 2$,

(ii-1) *$\tau = n$, $\phi(X)$ is a point, X is a quadric hypersurface $\mathbb{Q}^n$ in $\mathbb{P}^{n+1}$, and $L = \mathcal{O}_{\mathbb{Q}^n}(1)$;*

(ii-2) *$\tau = n$, X is a $\mathbb{P}^{n-1}$-bundle over a smooth projective curve W, and $L_F = \mathcal{O}_{\mathbb{P}^{n-1}}(1)$ for any fiber $F = \mathbb{P}^{n-1}$ of ϕ;*

(ii-3) *$\tau = 3/2$, $\phi(X)$ is a point, and $(X, L) = (\mathbb{P}^2, \mathcal{O}_{\mathbb{P}^2}(2))$.*

For $n \geq 3$,

(iii-1) *$\tau = n - 1$, $\phi(X)$ is a point, and $K_X + (n - 1)L = \mathcal{O}_X$;*

(iii-2) *$\tau = n - 1$, W is a smooth projective curve, and any fiber F of ϕ is a quadric hypersurface in $\mathbb{P}^n$ with $L_F = \mathcal{O}_F(1)$;*

(iii-3) *$\tau = n - 1$, X is a $\mathbb{P}^{n-2}$-bundle over a smooth projective surface W, and $L_F = \mathcal{O}_{\mathbb{P}^{n-2}}(1)$ for any fiber $F = \mathbb{P}^{n-2}$ of ϕ;*

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