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A REFINEMENT OF STEIN FACTORIZATION AND DEFORMATIONS OF SURJECTIVE MORPHISMS*

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Abstract. This paper is concerned with a refinement of the Stein factorization, and with applications to the study of deformations of morphisms. We show that every surjective morphism $f : X \to Y$ between normal projective varieties factors canonically via a finite cover of Y that is éale in codimension one. This "maximally étale factorization" satisfies a strong functorial property.

It turns out that the maximally étale factorization is stable under deformations, and naturally decomposes an étale cover of the Hom-scheme into a torus and into deformations that are relative with respect to the rationally connected quotient of the target Y. In particular, we show that all deformations of f respect the rationally connected quotient of Y.

Key words. Stein factorization, deformations of morphisms, maximal étale factorization, Hom-scheme.

AMS subject classifications. 14D15, 14J40, 14E99

1. Introduction and statement of results.

Throughout this paper, we consider surjective morphisms between algebraic varieties and their deformations. To fix notation, we use the following assumption.

Assumption 1.1. *$f : X \to Y$ will always denote a surjective holomorphic map between normal complex-projective varieties.*

The main method that we introduce is a refinement of the Stein factorization: we show that f factors canonically via a finite cover of Y that is étale in codimension one. This "maximally étale factorization" satisfies a strong functorial property which is defined in Section 1.1 below and turns out to be stable under deformations of f.

We employ the maximally étale factorization for a study of the deformation space $\mathrm{Hom}(X, Y)$ and show that an étale cover of the Hom-scheme naturally decomposes into a torus and into deformations that are relative with respect to the maximally rationally connected fibration of the target Y. In particular, we show that all deformations of f respect the rationally connected quotient of Y.

These result are summarized and properly formulated below.

1.1. The maximally étale factorization.

Under the Assumptions 1.1, suppose that there exists a factorization f,

$$X \xrightarrow{\alpha} Z \xrightarrow{\beta} Y, \quad f = \beta \circ \alpha \tag{1.1.1}$$

where β is finite and étale in codimension 1, i.e. étale outside a set of codimension ≥ 2.

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