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A RIGIDITY RESULT FOR p-DIVISIBLE FORMAL GROUPS*

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1. Introduction.

In this article we prove a rigidity result for p-divisible formal groups; see Thm. 4.3 for the statement. An important special case is the following. Consider a formal torus T over an algebraically closed field k of characteristic $p > 0$. Suppose $Z \subseteq T$ is an irreducible closed formal subscheme of T which is stable under the endomorphism $[1 + p^n]_T$ for some $n \geq 2$, where $[1 + p^n]_T : T \to T$ denotes "multiplication by $1 + p^n$" on the formal torus T. Then 4.3 asserts that Z is a formal subtorus of T.

If one assumes that k is equal to the algebraic closure $\overline{\mathbb{F}_p}$ of the prime field $\mathbb{F}_p$ and the closed formal subscheme Z in Thm. 4.3 is formally smooth over k, then the proof of 4.3 can be simplified. Section 2 contains lemmas in commutative algebra used to remove the extra assumptions above. For instance a weak desingularization result Prop. 2.1 for complete local integral domains over k with residue field k is used to remove the smoothness assumption on Z. The main tool for the proof of 4.3 is Prop. 3.1, a result on power series. The proof of Prop. 3.1 is elementary, so this article has the flavor of an excursion in "high school algebra" in the sense of Abhyankar.

The motivation of this article comes from the Hecke orbit problem for the reduction of a Shimura variety in characteristic p. See Conj. 6.2 in [10] for a statement of the conjecture for Siegel modular varieties, and [3] for a survey of the Hecke orbit problem and a sketch of a proof of the Hecke orbit conjecture for the Siegel modular varieties; see also [4]. The rigidity result 4.3 in this article, when combined with the theory of canonical coordinates on leaves in [6], allows one to linearize the Hecke orbit problem and reduce it to a question on global p-adic monodromy; see [3], [4]. See also [7, §6, §9] for an exposition of this linearization procedure in the case of ordinary abelian varieties. Thm. 4.3 has also been used in Hida's recent works [9] on the Iwasawa μ-invariant for p-adic L-functions; see §3 of [9].

In the present set-up, the statement of Thm. 4.3 appears to be in its optimal form. On the other hand one expects that 4.3 can be generlized and adapted to the situation of canonical coordinates for leaves, where the ambient formal scheme has, instead of a group structure, a *cascade* structure in the sense of B. Moonen. We hope to address this point in the near future.

2. Lemmas in commutative algebra.

PROPOSITION 2.1. *Let k be an algebraically closed field. Let R be a topologically finitely generated complete local domain over k. In other words, R is isomorphic to a quotient $k[[x_1, \ldots, x_n]]/P$, where P is a prime ideal of the power series*

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