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# PARAMETRIZATION OF SING Θ FOR A FANO 3-FOLD OF GENUS 7 BY MODULI OF VECTOR BUNDLES*

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**Abstract.** According to Mukai, any prime Fano threefold $X$ of genus 7 is a linear section of the spinor tenfold in the projectivized half-spinor space of Spin(10). The orthogonal linear section of the spinor tenfold is a canonical genus-7 curve $\Gamma$, and the intermediate Jacobian $J(X)$ is isomorphic to the Jacobian of $\Gamma$. It is proven that, for a generic $X$, the Abel-Jacobi map of the family of elliptic sextics on $X$ factors through the moduli space of rank-2 vector bundles with $c_1 = -K_X$ and $\deg c_2 = 6$ and that the latter is birational to the singular locus of the theta divisor of $J(X)$.

**Key words.** Spinors, spinor variety, Fano variety, moduli of vector bundles, intermediate Jacobian, Brill–Noether locus, orthogonal Grassmannian, theta divisor, elliptic sextic, symmetric powers of a curve

**AMS subject classifications.** 14J30

**0. Introduction.** This work is a sequel to the series of papers on moduli spaces $M_X(2; k, n)$ of stable rank-2 vector bundles on Fano 3-folds $X$ with Picard group $\mathbb{Z}$ for small Chern classes $c_1 = k,\ c_2 = n$. The nature of the results depends strongly on the index of $X$, which is defined as the largest integer that divides the canonical class $K_X$ in Pic $X$. Historically, the first Fano 3-fold for which the geometry of such moduli spaces was studied was the projective space $\mathbb{P}^3$, the unique Fano 3-fold of index 4. The most part of results for $\mathbb{P}^3$ concerns the problems of rationality, irreducibility or smoothness of the moduli space, see [Barth-1], [Barth-2], [Ha], [HS], [LP], [ES], [HN], [M], [BanM], [GS], [K], [KO], [CTT] and references therein.

The next case is the 3-dimensional quadric $Q^3$, which is Fano of index 3. Much less is known here, see [OS]. Further, the authors of [SW] identified the moduli spaces $M_X(2; -1, 2)$ on all the Fano 3-folds $X$ of index 2 except for the double Veronese cone $V_1'$, which are (in the notation of Iskovskikh) the quartic double solid $V_2$, a 3-dimensional cubic $V_3$, a complete intersection of two quadrics $V_4$, and a smooth 3-dimensional section of the Grassmannian $G(2, 5)$ by three hyperplanes $V_5$. It turns out that all the vector bundles in $M_X(2; -1, 2)$ for these threefolds are obtained by Serre's construction from conics. Remark that for $\mathbb{P}^3$ and $Q^3$ all the known moduli spaces are either rational or supposed to be rational, whilst [SW] provides first nonrational examples.

We will also mention the paper [KT] on the moduli of stable vector bundles on the flag variety $\mathbb{F}(1, 2)$, though it is somewhat apart, for $\mathbb{F}(1, 2)$ has Picard group $2\mathbb{Z}$. This is practically all what was known on the subject until the year 2000, when a new tool was brought into the study of the moduli spaces: the Abel–Jacobi map to the intermediate Jacobian $J(X)$. For the 3-dimensional cubic $X = V_3$, it was proved in [MT-1], [IM-1] that the open part of $M_X(2; 0, 2)$ parametrizing the vector bundles obtained by Serre's construction from elliptic quintics is sent by the Abel–Jacobi map isomorphically onto an open subset of $J(X)$. Druel [D] proved the irreducibility of

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