FINITE JET DETERMINATION OF LOCAL CR AUTOMORPHISMS THROUGH RESOLUTION OF DEGENERACIES*

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Dedicated to M. Salah Baouendi on the occasion of his seventieth birthday

Abstract. Let $M \subset \mathbb{C}^N$ be a connected real-analytic hypersurface whose Levi form is nondegenerate at some point. We prove that for every point $p \in M$, there exists an integer $k = k(M, p)$ such that germs at p of local real-analytic CR automorphisms of M are uniquely determined by their k-jets (at p). To prove this result we develop a new technique that can be seen as a resolution of the degeneracies of M. This procedure consists of blowing up M near an arbitrary point $p \in M$ regardless of its minimality or nonminimality; then, thanks to the blow-up, the original problem can be reduced to an analogous one for a very special class of nonminimal hypersurfaces for which one may use known techniques to prove the finite jet determination property of its CR automorphisms.

Key words. Finite jet determination, CR automorphism, blow-up, nonminimal hypersurface

AMS subject classifications. 32H02, 32H12, 32V05, 32V15, 32V20, 32V35, 32V40

1. Introduction. This paper is concerned with the finite jet determination problem for germs of CR automorphisms of real-analytic hypersurfaces in complex space. Our main motivation is the following conjecture that essentially goes back to the recent work of Baouendi, Ebenfelt and Rothschild [1]:

CONJECTURE 1.1. *Let $M \subset \mathbb{C}^{n+1}$ be a connected real-analytic holomorphically nondegenerate hypersurface, $n \geq 1$. Then for every $p \in M$, there exists a positive integer $k = k(M, p)$ such that germs at p of local real-analytic CR automorphisms of M are uniquely determined by their k-jets at p.*

Let us recall that a (connected) holomorphically nondegenerate real hypersurface is a real hypersurface for which there is no germ of a nontrivial holomorphic vector field tangent to an open piece of M (this notion was introduced by Stanton [32]). A solution to the above conjecture would provide a completely satisfactory local CR version for real-analytic hypersurfaces of $\mathbb{C}^{n+1}$ of the classical uniqueness theorem of H. Cartan [11] stating that holomorphic self-automorphisms of bounded domains in $\mathbb{C}^{n+1}$ are uniquely determined by their 1-jet at any point of the source domain. Indeed, holomorphic nondegeneracy appears to be the "natural" obstruction to finite jet determination, the necessity of the condition being observed in [1].

Much progress has been made in recent years toward the solution to the above mentioned conjecture and a number of important cases have been settled. Historically, the first case considered was when the given hypersurface has everywhere nondegenerate Levi-form. This was solved by E. Cartan [9, 10], Tanaka [33] and Chern-Moser [12], as a consequence of their solution to the biholomorphic equivalence problem. Furthermore, in that setting unique determination by 2-jets holds at every point.