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MANIFOLDS OF HOLOMORPHIC MAPPINGS FROM STRONGLY PSEUDOCONVEX DOMAINS*

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Dedicated to Professor Salah Baouendi on the occasion of his seventieth birthday

Abstract. Let D be a bounded strongly pseudoconvex domain in a Stein manifold, and let Y be a complex manifold. We show that many classical spaces of maps $\bar{D} \to Y$ which are holomorphic in D are infinite dimensional complex manifolds which are modeled on locally convex topological vector spaces (Banach, Hilbert or Fréchet). This holds in particular for Hölder and Sobolev spaces of holomorphic maps.

Key words. Stein manifolds, strongly pseudoconvex domains, manifolds of holomorphic mappings

AMS subject classifications. 32E10, 32E30, 32H02, 46G20, 46T10, 54C35, 58B12, 58D15

1. Introduction. Given complex manifolds (or complex spaces) X and Y, it is a natural question whether the space $\mathcal{H}(X,Y)$ of all holomorphic mappings $X \to Y$ is also a complex manifold (resp. a complex space). If X is a compact complex space without boundary then $\mathcal{H}(X,Y)$ is a finite dimensional complex space which can be identified with an open subset in the Douady space $\mathcal{D}(X \times Y)$, [2, Theorem 1.5]. The set of all holomorphic maps from a noncompact manifold in general does not admit any particularly nice structure.

In §2 of this paper we prove the following results.

THEOREM 1.1. *Let D be a relatively compact strongly pseudoconvex domain in a Stein manifold, and let Y be a complex manifold.*

(i) The Hölder space $\mathcal{A}^{k,\alpha}(D,Y) = \mathcal{C}^{k,\alpha}(\bar{D},Y) \cap \mathcal{H}(D,Y)$ is a complex Banach manifold for every $k \in \mathbb{Z}_+$ and $0 \le \alpha < 1$.

(ii) $\mathcal{A}^\infty(D,Y) = \mathcal{C}^\infty(\bar{D},Y) \cap \mathcal{H}(D,Y)$ is a complex Fréchet manifold.

(iii) The Sobolev space $L^{k,p}_{\mathcal{O}}(D,Y) = L^{k,p}(\bar{D},Y) \cap \mathcal{H}(D,Y)$ is a complex Banach manifold for $k \in \mathbb{N}$, $p \ge 1$ and $kp > \dim_{\mathbb{R}} D$ (resp. a complex Hilbert manifold if $p = 2$).

If $L(D,Y)$ denotes any of the above manifolds of maps then the tangent space $T_f L(D,Y)$ at a point $f \in L(D,Y)$ is $L_h(D, f^*TY)$, the space of sections of class $L(D)$ of the complex vector bundle $h\colon f^*TY \to \bar{D}$. If D is contractible, or if $\dim D = 1$, then $T_f L(D,Y) \approx L(D,\mathbb{C}^m)$ with $m = \dim Y$.

The analogous conclusions hold if $\bar{D}$ is a compact complex manifold with Stein interior D and smooth strongly pseudoconvex boundary bD; according to Heunemann [21] and Ohsawa [31] (see also Catlin [4]) such $\bar{D}$ embeds as a smoothly bounded strongly pseudoconvex domain in a Stein manifold.

The special case of Theorem 1.1 (i) with $\alpha = 0$ was proved recently in [11].

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