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# EXCEPTIONAL BLOWUP SOLUTIONS TO QUASILINEAR WAVE EQUATIONS II*

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**1. Introduction.** This Note is a continuation of the paper "Exceptional Blowup Solutions to Quasilinear Wave Equations" [1]. In this previous paper, we constructed, for some quasilinear wave equations, solutions blowing up at the origin like $t^{-2}$, which we considered to be an exceptional rate (the standard one in this context being $t^{-1}$, see [6]). We were encouraged by questions of the Referee (whom we thank) to investigate more precisely the stability of such solutions (an issue vaguely touched upon in [1]). It turns out that, depending on the perturbation of the Cauchy data, we can make the singularity of the solution either disappear, or go back to the generic $t^{-1}$ case.

Since this paper is dedicated to M. S. Baouendi, we are happy to underline the similarity in spirit between previous constructions of conterexamples [7], [8], and the present work : in both cases, the insight is obtained through a careful self-contained construction.

**2. Notation and main result.** The notation and the framework is the same as in [1]. For simplicity, we restrict our attention to $n = 2$, and do not handle the 1D case (though it is straightforward). Thus the variables and dual variables are

$$x = (x_1, x_2, x_3), y = x_2, t = x_3, \xi = (\xi_1, \xi_2, \xi_3), \eta = \xi_2, \tau = \xi_3.$$

We consider a quasilinear wave equation with real *analytic* coefficients

$$P(u) = \Sigma p_{ij}(\partial u)\partial^2_{ij}u = 0, p_{ij} = p_{ji}, p_{3,3} = 1.$$

We denote here

$$\partial u = (\partial_1 u, \partial_2 u, \partial_3 u), p(\partial u; \xi) = \Sigma p_{ij}(\partial u)\xi_i\xi_j.$$

We assume given a point $(\overline{\partial u}, \bar{\xi})$ where

$$p(\overline{\partial u}; \bar{\xi}) = 0, (\partial_\tau p)(\overline{\partial u}; \bar{\xi}) \neq 0, \bar{\xi}_1 = -1,$$

and the frozen operator $\Sigma p_{ij}(\overline{\partial u})\partial^2_{ij}$ is strictly hyperbolic with respect to $t$. Noting $D_j p = \partial_{(\partial_j u)} p$, we assume moreover that the given point is *linearly degenerate*, that is

$$(\bar{\xi}.D)p(\overline{\partial u}; \bar{\xi}) = 0.$$

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