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PAIRS OF INVOLUTIONS OF GLANCING HYPERSURFACES*

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Dedicated to Salah M. Baouendi on the occasion of his seventieth birthday

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1. Introduction. Let ω be the standard symplectic 2-form on $\mathbf{R}^{2n}$, given by

$$\omega = \sum_{j=1}^{n} d\xi_j \wedge dx_j, \quad (\xi, x) = (\xi_1, \cdots, \xi_n, x_1, \cdots, x_n) \in \mathbf{R}^{2n}.$$

Consider two real analytic hypersurfaces in $\mathbf{R}^{2n}(n \geq 2)$ defined by

$$F : f(\xi, x) = 0, \quad G : g(\xi, x) = 0,$$

where f, g are real analytic functions. F and G are said to be *glancing* at $p \in F \cap G$ if

$$\{f, g\}(p) = 0, \quad df \wedge dg\,(p) \neq 0,$$
$$\{f, \{f, g\}\}(p) \neq 0 \neq \{g, \{g, f\}\}(p),$$

in which $\{f, g\}$ is the Poisson bracket of f, g with respect to ω, defined by

$$\{f, g\} = X_f g, \quad X_f = \sum \frac{\partial f}{\partial x_j}\frac{\partial}{\partial \xi_j} - \frac{\partial f}{\partial \xi_j}\frac{\partial}{\partial x_j}.$$

A (local) map from $\mathbf{R}^{2n}$ to $\mathbf{R}^{2n}$ is said to be symplectic if it preserves ω. Given two pairs of hypersurfaces $\{F_j, G_j\}$ glancing at $p_j (j = 1, 2)$ respectively, they are equivalent if there exists a real analytic symplectic mapping ϕ defined near p_1 such that

$$\phi(p_1) = p_2, \quad \phi(F_1) = F_2, \quad \phi(G_1) = G_2.$$

Since we consider local equivalence only, we assume that $p_1 = p_2 = 0$.

In [5], Melrose showed that each pair of glancing smooth hypersurfaces in $\mathbf{R}^{2n}(n \geq 2)$ is equivalent to the pair

$$\widehat{F} : x_1 = 0, \quad \widehat{G} : \xi_2 = \xi_1^2 + x_1 \tag{1.1}$$

under a (C^∞) smooth change of coordinates; Melrose's argument also shows that all real analytic glancing hypersurfaces are equivalent to the above normal form by formal symplectic maps. It was proved by Oshima [6] for $n \geq 3$ and by the second author [3] for $n \geq 2$ that for some pairs of real analytic glancing hypersurfaces, the normal form cannot be achieved by any convergent symplectic map.

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