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EDGE OF THE WEDGE THEORY IN INVOLUTIVE STRUCTURES*

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Abstract. This paper describes the C^∞ wave-front set of the boundary values of approximate solutions in wedges $\mathcal{W}$ of involutive structures $(M, \mathcal{V})$ that are not necessarily locally integrable. It is shown that the C^∞ wave-front set of the boundary value is contained in the polar of a certain cone $\Gamma^T(\mathcal{W})$ contained in $\Re\mathcal{V} \cap TX$ where X is a maximally real edge of $\mathcal{W}$. A converse result is also established.

Key words. Involutive, Maximally real, Wedge, Edge, Wave-front, approximate solution

AMS subject classifications. Primary 35F15, 35B30, 42B30; Secondary 42A38, 30E25

1. Introduction. Let M be a C^∞ manifold and $\mathcal{V} \subseteq \mathbb{C}TM$ a subbundle of rank n which is involutive, that is, the bracket of two smooth sections of $\mathcal{V}$ is also a section of $\mathcal{V}$. We will refer to the pair $(M, \mathcal{V})$ as an involutive structure. The involutive structure $(M, \mathcal{V})$ is called locally integrable if the orthogonal of $\mathcal{V}$ in $\mathbb{C}T^*M$ is locally generated by exact forms. In [EG] assuming that $(M, \mathcal{V})$ is locally integrable, the authors proved some microlocal regularity results for a distribution u on certain submanifolds E of M where u arises as the boundary value of a solution on a wedge $\mathcal{W}$ in M with edge E. These results were expressed in terms of the hypo-analytic wave-front set developed in [BCT]. In this article we prove some analogous results in the setting of involutive structures that are not necessarily locally integrable, and for boundary values of approximate solutions (Definition 2.4) in wedges.

In section 2 we summarize some of the notions from [EG] that we need to state our main results, Theorems 3.1 and 3.2. Section 3 is devoted to the proofs of these results. Finally, in section 4 we present a sufficient condition for the existence of boundary values that is used in the proof of Theorem 3.2.

2. Preliminaries. In this section we will briefly recall some of the notions and results we will need about involutive structures. The reader is referred to [EG] for more details.

We assume $(M, \mathcal{V})$ is an involutive structure and the fiber dimension of $\mathcal{V}$ equals n. A distribution f on M is called a solution if $Lf = 0$ for all smooth sections L of $\mathcal{V}$. A real cotangent vector $\sigma \in T_p^*M$ is said to be characteristic for the involutive structure $(M, \mathcal{V})$ if $\sigma(L) = 0$ for all $L \in \mathcal{V}_p$ and we let

$$T_p^0 = \{\sigma \in T_p^*M : \sigma \text{ is characteristic for } (M, \mathcal{V})\}.$$

Even when $\mathcal{V}$ is a line bundle, the dimension of T_p^0 may not be constant as p varies. However, when $\mathcal{V}$ is a CR structure, that is, $\mathcal{V} \cap \overline{\mathcal{V}} = \{0\}$, then T^0 is a vector bundle.

DEFINITION 2.1. *A smooth submanifold X of M is called maximally real if $\mathbb{C}T_pM = \mathcal{V}_p \oplus \mathbb{C}T_pX$ for each $p \in X$.*

If X is a maximally real submanifold and $p \in X$, define

$$\mathcal{V}_p^X = \{L \in \mathcal{V}_p : \Re L \in T_pX\}.$$

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