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BUILDINGS AND THEIR APPLICATIONS IN GEOMETRY AND TOPOLOGY*

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To the memory of Professor S.S. Chern

Abstract. Buildings were first introduced by J. Tits in 1950s to give systematic geometric interpretations of exceptional Lie groups and have been generalized in various ways: Euclidean buildings (Bruhat-Tits buildings), topological buildings, $\mathbb{R}$-buildings, in particular $\mathbb{R}$-trees. They are useful for many different applications in various subjects: algebraic groups, finite groups, finite geometry, representation theory over local fields, algebraic geometry, Arakelov intersection for arithmetic varieties, algebraic K-theories, combinatorial group theory, global geometry and algebraic topology, in particular cohomology groups, of arithmetic groups and S-arithmetic groups, rigidity of cofinite subgroups of semisimple Lie groups and nonpositively curved manifolds, classification of isoparametric submanifolds in $\mathbb{R}^n$ of high codimension, existence of hyperbolic structures on three dimensional manifolds in Thurston's geometrization program. In this paper, we survey several applications of buildings in differential geometry and geometric topology. There are four underlying themes in these applications:

1. Buildings often describe the geometry at infinity of symmetric spaces and locally symmetric spaces and also appear as limiting objects under degeneration or scaling of metrics.
2. Euclidean buildings are analogues of symmetric spaces for semisimple groups defined over local fields and their discrete subgroups.
3. Buildings of higher rank are rigid and hence objects which contain or induce higher rank buildings tend to be rigid.
4. Additional structures on buildings, for example, topological buildings, are important in applications for infinite groups.

Key words. buildings, Tits building, Bruhat-Tits building, spherical building, Euclidean building, R-building, rigidity of lattices, super-rigidity, S-arithmetic groups

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Contents

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