

CHERN'S WORK IN GEOMETRY*

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It's fair to say that E. Cartan is the grandfather of differential geometry and S.S. Chern is the father of modern differential geometry.

Together they have created a beautiful and rich subject that have reached out to every branch of mathematics and physics.

Right before he died, Chern said that he is going to see the great Greek geometers. There is no doubt that he had reached the same status as these great geometers.

Now, we would like to review the major events in the glorious history of geometry.

Pythagoras (524-480 B.C.) found the Pythagoras theorem for triangles.

Euclid (325-265 B.C.) formulated axioms for Euclidean geometry.

Archimedes (287-212 B.C.) initiated the use of infinite processes and launched the study of conics.

Descartes (1596-1650) introduced coordinates, this is the birth of analytic geometry where algebra and geometry are merged.

G. Desargues (1591-1661) invented projective geometry.

Fermat (1601-1665) shaped variational principle when studying optics.

Newton (1642-1727) and Leibniz(1646-1716) independently created the great calculus.

Euler (1707-1783) studied combinational geometry and developed the method of calculus of variation.

Gauss (1777-1885) pioneered the study of intrinsic geometry.

Riemann (1826-1866) announced Riemannian geometry in 1854 in his Habilitationsschrift.

Sophus Lie (1842-1899) created the theory of transformation groups and discovered contact geometry.

F. Klein (1849-1925) announced the Erlangen programm which defines geometry as the study of a space with a group of transformation in 1872.

The group of projective collineations is the most encompassing group and the resulting geometry is projective geometry. Contributors include: J.V. Poncelet (1788-1867), A.F. Möbius (1790-1868), M. Chasles (1793-1880), J. Steiner (1796-1863).

There are many other geometries, such as affine geometry and conformal geometry whose corresponding groups are respectively the affine group and the conformal group.

A. Weil wrote in his preface to *Selected Papers of S.S. Chern*:

“The psychological aspects of true geometric intuition will perhaps never be cleared up. . . . Whatever the truth of the matter, mathematics in our century would not have made such impressive progress without the geometric sense of Elie Cartan, Heinz Hopf, Chern and a very few more. It seems safe to predict that such men will always be needed if mathematics is to go on as before.”

*This is the speech given on the Harvard Memorial Conference for S.S. Chern.

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