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003

VANISHING OF TOP EQUIVARIANT CHERN CLASSES OF REGULAR EMBEDDINGS*

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1. Introduction. Let G be a connected affine algebraic group and let X be a regular G-variety in the sense of [BDP] (recalled in Definition 2.2 below). The variety X contains an open orbit G/H whose complement D is a strictly normal crossing divisor in X. In this note we show the following vanishing result for rational equivariant Chern classes of the bundle of logarithmic differentials on the variety X:

$$c_i^G(\Omega_X^1(\log D)) = 0 \quad \text{for } i > \dim(X) - \mathrm{rk}(G) + \mathrm{rk}(H).$$

The motivation for this vanishing result originated in the second author's interest in a higher rank generalization of Gieseker's proof [G] of the Newstead-Ramanan conjecture. The conjecture (or rather its higher rank generalization) says that for coprime r and d and for $g \geq 1$ the Chern classes of the tangent bundle of the moduli space of stable vector bundles of rank r and degree d on a curve of genus g vanish in degrees larger than $r(r-1)(g-1)$ (cf. [EK], bottom of page 844).

In order to explain the relationship between this conjecture and the result proven here let us first sketch Gieseker's degeneration of moduli spaces of vector bundles.

Let B a smooth curve, which serves as the base scheme of the degeneration. Let $\mathcal{X} \to B$ be a proper flat family of algebraic curves of genus $g \geq 2$ which is smooth outside a point $x \in B$. Assume that the fiber X over x is irreducible with a unique singular point p which is an ordinary double point. Let $\tilde{X}$ be the normalization of X. Let r be a positive integer and let d be an integer prime to r. Then there exists a variety $M(\mathcal{X}/B)$ proper and flat over B such that

- the fiber of $M(\mathcal{X}/B) \to B$ over any point $y \in B \setminus \{x\}$ is the moduli space $M(Y)$ of stable vector bundles of rank r and degree d on the curve $Y = \mathcal{X}_y$,
- the variety $M(\mathcal{X}/B)$ is nonsingular and its fiber $M(X)$ over x is a normal crossing divisor in $M(\mathcal{X}/B)$.
- Let $M(\tilde{X})$ be the moduli space of rank r degree d vector bundles on the curve $\tilde{X}$. Then there is a principal $\mathrm{GL}_r \times \mathrm{GL}_r$-bundle P on $M(\tilde{X})$, and a smooth $\mathrm{GL}_r \times \mathrm{GL}_r$-equivariant compactification of $\mathrm{GL}_r = (\mathrm{GL}_r \times \mathrm{GL}_r)/\operatorname{diag}(\mathrm{GL}_r)$ which we denote by KGL_r, such that the normalization $\tilde{M}$ of $M(X)$ is birationally equivalent to the locally trivial KGL_r-fibration

$$f : M' := P \times^{\mathrm{GL}_r \times \mathrm{GL}_r} \mathrm{KGL}_r \to M(\tilde{X})$$

associated to the principal bundle $P \to M(\tilde{X})$.

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