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CORRECTION TO "THE SEIBERG-WITTEN EQUATIONS FOR FAMILIES AND DIFFFEOMORPHISMS OF 4-MANIFOLDS", ASIAN J. MATH., VOL. 7, NO. 1, 133–138, 2003*

NOBUHIRO NAKAMURA†

Proposition 2.1 in [1] is incorrect. We need the extra assumption

$$H^1(B;\mathcal{H}^1)=0, \tag{0.1}$$

where $\mathcal{H}^1$ is the Serre local system of $H^1(X;\mathbb{Z}_2)$: In the proof of Proposition 2.1 in [1] two exact sequences are stated. Under the assumption $b_1(X)=0$, the exact sequence with $\mathbb{Z}$-coefficient is valid, since $E_2^{p,1}=0$ for all p. However, the exact sequence with $\mathbb{Z}_2$-coefficient does not hold in general. Alternatively, under (0.1), we have

$$H^1(X;\mathbb{Z}_2)^{\pi_1(B)} \xrightarrow{d_2} H^2(B;\mathbb{Z}_2)\to H^2(\mathbb{X};\mathbb{Z}_2)\to \operatorname{Ker} d_3,$$

which, together with the $\mathbb{Z}$-coefficient sequence, implies the proposition.

Accordingly, Theorem 1.1, Theorem 1.2 and Corollary 1.3 in [1] are to be modified as follows.

(1) When $B=S^1$ and $\mathbb{X}$ is a mapping cylinder $X\times[0,1]/f$, (0.1) is equivalent to the condition

$$H^1(X;\mathbb{Z}_2)^{f^*}=0, \tag{0.2}$$

where f^* is the automorphism on $H^1(X;\mathbb{Z}_2)$ induced from f. In fact, $H^1(B;\mathcal{H}^1)\cong \operatorname{Coker}(f^*-I)$, where I is the unit matrix. Hence we need the extra assumption (0.2) in Theorem 1.2.

(2) Instead of Corollary 1.3 in [1], an alternative example of Theorem 1.2 is given by a connected sum of an Enriques surface and a rational homology 4-sphere Y with $H^1(Y;\mathbb{Z}_2)=\mathbb{Z}_2$.

(3) When $B=T^2$ and $\mathbb{X}$ is a X-bundle made from two diffeomorphism f and g, (0.1) is equivalent to the condition

$$\operatorname{rank}\begin{pmatrix} I-g^* \\ -(I-f^*)\end{pmatrix}=\dim H^1(X;\mathbb{Z}_2), \tag{0.3}$$

where f^* and g^* are automorphisms on $H^1(X;\mathbb{Z}_2)$ induced from f and g. Hence we need the extra assumption (0.3) in Theorem 1.1.

REFERENCES

[1] N. NAKAMURA, *The Seiberg-Witten equations for families and diffeomorphisms of 4-manifolds*, Asian J. Math., 7 (2003), pp. 133–138.

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†Research Institute for Mathematical Sciences, Kyoto university, Kyoto, 606-8502, Japan (nakamura@kurims.kyoto-u.ac.jp).