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ABSTRACT PERIODIC HAMILTONIAN SYSTEMS

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1. Introduction. This work concerns existence via variational arguments for abstract Hamiltonian systems

$$\begin{aligned} \mathcal{A}^* p &\in \partial_y \mathcal{H}(y,p) + f \\ \mathcal{A} y &\in \partial_p \mathcal{H}(y,p) + g \end{aligned} \tag{1.1}$$

in the product Hilbert space $\mathcal{X} \times \mathcal{X}$, where $\mathcal{H} : \mathcal{X} \times \mathcal{X} \to R$ is a convex continuous function, $\partial \mathcal{H} = (\partial_y \mathcal{H}, \partial_p \mathcal{H})$ is its subdifferential, $\mathcal{A} : D(\mathcal{A}) \subset \mathcal{X} \to \mathcal{X}$ is a linear, densely defined closed operator with closed range $R(\mathcal{A})$ and $\mathcal{A}^*$ is its adjoint; f, g are fixed elements of $\mathcal{X}$.

A prototype of this system is the periodic Hamiltonian system

$$\begin{aligned} & y'(t) + Ay(t) \in \partial_p H(y(t), p(t)) + g(t), \quad t \in (0,T) \\ & p'(t) - A^* p(t) \in -\partial_y H(y(t), p(t)) - f(t) \\ & y(0) = y(T), \quad p(0) = p(T), \end{aligned} \tag{1.2}$$

where $A : D(A) \subset X \to X$ is the infinitesimal generator of a C_0-semigroup on the Hilbert space X, $H : X \times X \to R$ is a continuous convex function and $f, g \in L^2(0,T;X)$. Other problems which can be written in this form and will be considered below are second-order Hamiltonian systems with periodic conditions and elliptic Hamiltonian systems. The variational approach to be used below in existence theory of system (1.1) is inspired by the duality theory developed for finite-dimensional periodic Hamiltonian systems by Clarke and Ekeland ([6]). In a preliminary form some of these results were presented in [2].

Throughout in the sequel we assume familiarity with basic results and concepts of convex analysis and infinite-dimensional convex optimization (see e.g. [1]).

2. The main existence result. Here $\mathcal{X}$ is a real Hilbert space with the norm denoted $\|\cdot\|$ and the scalar product $\langle \cdot, \cdot \rangle$.

Hypotheses:

(i) *$\mathcal{H} : \mathcal{X} \times \mathcal{X} \to R$ is convex, continuous and satisfies the growth condition*

$$\gamma_1 \|y\| + \gamma_2 \|p\| + C_1 \le \mathcal{H}(y,p) \le \omega(\|y\|^2 + \|p\|^2) + C_2, \quad \forall\, (y,p) \in \mathcal{X} \times \mathcal{X} \tag{2.1}$$

where $\gamma_1, \gamma_2, \omega > 0$.

(ii) *$\mathcal{A} : \mathcal{X} \to \mathcal{X}$ is a linear, densely defined closed operator with closed range $R(\mathcal{A})$.*

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