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MAXIMAL ATTRACTOR AND INERTIAL SETS FOR A CONSERVED PHASE FIELD MODEL

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1. Introduction. In this paper we consider the following conserved phase field model proposed by Caginalp ([7]):

$$P_0 \quad \begin{cases} \tau\varphi_t = -\xi^2\Delta(\xi^2\Delta\varphi - g(\varphi) + 2u) & \text{in } \Omega\times\mathbb{R}^+, \\ u_t + \frac{\ell}{2}\varphi_t = K\Delta u & \text{in } \Omega\times\mathbb{R}^+, \\ \frac{\partial\varphi}{\partial n} = \frac{\partial\Delta\varphi}{\partial n} = \frac{\partial v}{\partial n} = 0 & \text{on } \partial\Omega\times\mathbb{R}^+, \\ \varphi(x,0) = \varphi_0(x),\ u(x,0) = u_0(x) & x\in\Omega, \end{cases}$$

where Ω is an open bounded set in $\mathbb{R}^n$, $n = 1, 2, 3$, with smooth boundary $\partial\Omega$. Here $g = G'$, where G is a double well potential for which $g(s) = \frac{1}{2}(s^3 - s)$ and the unknown functions u and φ denote respectively the temperature and the order parameter or phase field. The dimensionless temperature u is scaled so that $u = 0$ corresponds to the standard planar equilibrium melting temperature and φ is scaled so that φ near 1 corresponds to the liquid phase and φ near -1 corresponds to the solid phase. The interface between liquid and solid described by the phase field model has finite width and contains all points where φ vanishes. In fact, Problem P_0 can be viewed as an approximating problem for the Stefan problem with surface tension ([6, 7]).

The positive constants ℓ and K represent the dimensionless latent heat and the diffusivity respectivly. The positive constants τ and ξ represent a relaxation time and a correlation length.

Problem P_0 can be viewed as a conserved version of the standard second-order phase field equations. In the second-order version the internal energy $e = \int_\Omega (u + \frac{\ell}{2}\varphi)$ is conserved. In the fourth-order version both the internal energy e and the "total mass" $M = \int_\Omega \varphi$ are conserved quantities. For the sake of generality we assume in this paper that the function g has the slightly more general form

$$g(s) = \sum_{j=0}^{2p-1} a_j s^j \quad \text{with} \quad a_{2p-1} > 0,\ p \geq 2 \ \text{ if } \ n = 1, 2 \ \text{ and } \ p = 2 \ \text{ if } \ n = 3.$$

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