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BLOW UP OF CRITICAL AND SUBCRITICAL NORMS IN SEMILINEAR HEAT EQUATIONS

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1. Introduction. We consider the semilinear heat equation

$$\begin{aligned} &u_t - \Delta u = |u|^{p-1}u, \quad x \in \Omega, \ \ t \in [0,T], \\ &u(t,x) = 0, \quad x \in \partial\Omega, \ \ t \in [0,T], \\ &u(0,x) = u_0(x), \quad x \in \Omega, \end{aligned} \tag{1}$$

with $p > 1$ and $u_0 \in L^\infty(\Omega)$, where Ω is a smooth, bounded domain in $\mathbb{R}^N$ or $\Omega = \mathbb{R}^N$.

Let $u \in C((0,T_m) \times \overline{\Omega})$ be a solution of (1) which blows up in finite time, i.e., u is defined on the maximal interval of time $[0,T_m)$, with $T_m < +\infty$. By the blow up alternative, we know that u blows up in $L^\infty(\Omega)$, i.e., $\lim_{t\uparrow T_m} \|u(t)\|_{L^\infty(\Omega)} = +\infty$. A classical problem is to determine whether $\|u(t)\|_{L^q(\Omega)}$ also blows up as $t \uparrow T_m$, for some $q < +\infty$, and if so, at what rate.

We first note that the value $q = \frac{N(p-1)}{2}$ plays a critical role as in the problem of local existence for (1) with $u_0 \in L^q(\Omega)$. The case where q is supercritical, i.e., $q > \frac{N(p-1)}{2}$, is essentially known. Indeed,

$$\lim_{t\uparrow T_m} \|u(t)\|_{L^q(\Omega)} = +\infty,$$

for all $\frac{N(p-1)}{2} < q \leq +\infty$, $q \geq 1$. Moreover,

$$\liminf_{t\uparrow T_m} (T_m - t)^\delta \|u(t)\|_{L^q(\Omega)} > 0, \tag{2}$$

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