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LARGE-AMPLITUDE HIGH-FREQUENCY WAVES FOR QUASILINEAR HYPERBOLIC SYSTEMS

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1. Introduction

This paper is concerned with the existence and stability of *multidimensional large-amplitude high-frequency waves* associated to a linearly degenerate field. They are families $\{u^\varepsilon; \varepsilon \in (0,1]\}$ of solutions of a hyperbolic system of conservation laws on a fixed domain independent of ε, such that

$$u^\varepsilon(t,x) \underset{\varepsilon\to 0}{\sim} \mathbf{U}^\varepsilon\big(\,t,x,\vec{\boldsymbol{\varphi}}(t,x)/\varepsilon\,\big)\,, \qquad \partial_\theta U^\varepsilon(t,x,\theta) = O(1)\,. \tag{1.1}$$

These $O(1)$ rapid variations are anomalous oscillations in the general context of nonlinear geometric optics, where the standard regime concerns $O(\varepsilon)$ oscillations:

$$u^\varepsilon(t,x) \underset{\varepsilon\to 0}{\sim} u_0(t,x) + \varepsilon \mathbf{U}_1^\varepsilon\big(\,t,x,\vec{\boldsymbol{\varphi}}(t,x)/\varepsilon\,\big)\,. \tag{1.2}$$

However, when the oscillations are associated to linearly degenerate modes, the equations for $\mathbf{U}_1$ are linear, suggesting that, in this case, oscillations of larger amplitude can be considered.

A strong motivation for studying waves (1.1) is the existence of *simple waves* associated to linearly degenerate modes (see [20]). They are solutions of the form

$$\mathbf{V}\big(h(\mathbf{k}\cdot x - \omega\, t)\big), \tag{1.3}$$

with $\mathbf{V} \in \mathcal{C}^1(I;\mathbb{R}^N)$ and $(\omega,\mathbf{k}) \in \mathbb{R}^{1+d}$ suitably chosen, and h an arbitrary function in $\mathcal{C}^1(\mathbb{R};I)$. Fix any $h \in \mathcal{C}^1(\mathbb{R};I)$. The functions

$$u^\varepsilon(t,x) = \mathbf{U}\big(\varphi(t,x)/\varepsilon\big), \qquad \mathbf{U} = \mathbf{V}\circ h, \qquad \varphi(t,x) = \mathbf{k}\cdot x - \omega\, t \tag{1.4}$$

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