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ABOUT THE LOSS OF DERIVATIVES FOR STRICTLY HYPERBOLIC EQUATIONS WITH NON-LIPSCHITZ COEFFICIENTS

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1. Introduction

In the last years we have several papers devoted to the well-posedness of the strictly hyperbolic Cauchy problem

$$u_{tt} - \sum_{k,l=1}^{n} a_{kl}(t,x)u_{x_kx_l} = 0, \;\; u(0,x) = u_0(x),\; u_t(0,x) = u_1(x), \qquad (1.1)$$

in classes of solutions whose values are in Sobolev spaces, where we suppose additionally $a_{kl} = a_{lk}$. In further considerations we assume that the coefficients belong to $\mathcal{B}^\infty$ with respect to x. By $\mathcal{B}^\infty$ we denote the space of infinitely differentiable functions having bounded derivatives on $\mathbb{R}^n$. Its topology is generated by the family of norms of spaces $\mathcal{B}^p$, $p \geq 0$, consisting of functions with bounded derivatives up to order p. We obtain C^∞ well-posedness results from well-posedness results for Sobolev solutions together with the cone of dependence property. Thus, in this paper our goal is to derive energy inequalities for solutions which imply in a more or less standard way well-posedness for solutions in classes of solutions whose values are in Sobolev spaces, C^∞, respectively.

There are different ways to describe the non-Lipschitz behavior of coefficients with respect to t to get well-posedness of solutions whose values are in Sobolev classes. One way goes back to [4]. The authors supposed for the

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