

REDUCED MEASURES FOR OBSTACLE PROBLEMS

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1. INTRODUCTION

Let $\Omega \subset \mathbb{R}^N$, $N \geq 1$, be a smooth bounded domain. In this paper, we study the problem

$$\begin{cases} -\Delta u + \beta(u) \ni \mu & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \quad (1.1)$$

where μ is a finite measure in Ω and β is a maximal monotone graph (m.m.g.) such that $(0, 0) \in \text{graph } \beta$.

Throughout most of the Introduction, we assume that

$$\text{dom } \beta = (-\infty, a] \quad \text{for some } 0 \leq a < \infty \quad (1.2)$$

and

$$\beta(t) = \{0\} \quad \forall t \leq 0. \quad (1.3)$$

(However, the case where $\text{dom } \beta = [-b, a]$, $b \geq 0$, is also of interest and will be discussed at the end of the Introduction.) A typical example of a m.m.g. β satisfying (1.2)–(1.3) is the following

$$\beta(t) = \begin{cases} \{g(t)\} & \text{if } t < a, \\ [g(a), \infty) & \text{if } t = a, \\ \emptyset & \text{if } t > a, \end{cases}$$

where $g : (-\infty, a] \rightarrow [0, \infty)$ is any continuous nondecreasing function such that $g(t) = 0$, for all $t \leq 0$.

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