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FIRST EIGENVALUE AND MAXIMUM PRINCIPLE FOR FULLY NONLINEAR SINGULAR OPERATORS

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1. Introduction

In this paper we introduce the notion of *first eigenvalue* for fully nonlinear operators which are nonvariational but homogeneous. It is unnecessary to emphasize the importance of knowing the spectrum of a linear operator. When the operator is a uniformly elliptic operator of second order $Lu = tr(A(x)D^2u)$ associated with a Dirichlet problem in a bounded domain Ω the spectrum is a point spectrum bounded from below and the first eigenvalue $\bar\lambda$ is paramount. It is well known that $\bar\lambda$ is positive and it satisfies:

- There exists a positive function ϕ satisfying
$$\begin{cases} L\phi + \bar\lambda\phi = 0 & \text{in } \Omega \\ \phi = 0 & \text{on } \partial\Omega. \end{cases}$$
- For any $\lambda < \bar\lambda$ and for any $f \in L^N(\Omega)$ there exists a unique u such that
$$\begin{cases} Lu + \lambda u = f & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

See e.g. [13] for the proof of these results under suitable conditions on $A(x)$ and Ω. Berestycki, Nirenberg and Varadhan, in [3], have characterized the first eigenvalue of $-L$ in Ω by the fact that it is the supremum of the values λ such that $L+\lambda$ satisfies the maximum principle in Ω. Let us recall that $L+\lambda$ *satisfies the maximum principle in* Ω if any solution of $Lu+\lambda u \geq 0$ in Ω which is nonpositive on the boundary of Ω is nonpositive in Ω.

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