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LOCATION OF SPECTRUM AND STABILITY OF SOLUTIONS FOR MONOTONE PARABOLIC SYSTEM

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1. Introduction. We consider the parabolic system of equations

$$\frac{\partial u}{\partial t} = a\Delta u + F(u, x'), \tag{1.1}$$

with the boundary condition

$$\left.\frac{\partial u}{\partial \nu}\right|_{\partial\Omega} = 0, \tag{1.2}$$

where $u = (u_1, \ldots, u_n)$, $x = (x_1, \ldots, x_m) \in \Omega \subset R^m$, Ω is an infinite cylinder with the axis in the x_1-direction and with sufficiently smooth boundary $\partial\Omega$. The coordinates in the section of the cylinder are denoted by $x' = (x_2, \ldots, x_m)$. We suppose that a is a constant diagonal matrix with positive diagonal elements and function $F = (F_1, \ldots, F_n)$ satisfies the condition

$$\frac{\partial F_i}{\partial u_j} \geq 0, \quad i \neq j. \tag{1.3}$$

In this work we study local and global stability of travelling waves described by the problem (1.1), (1.2). We recall that a travelling wave solution is a solution of the form $u(x,t) = w(x_1 - ct, x_2, \ldots, x_m)$. Here c is a constant, the wave velocity. The function $w(x)$ is a stationary solution of the problem

$$\frac{\partial v}{\partial t} = a\Delta v + c\frac{\partial v}{\partial x_1} + F(v, x'), \quad \left.\frac{\partial v}{\partial \nu}\right|_{\partial\Omega} = 0. \tag{1.4}$$

As is known, local stability of travelling waves is determined by the location of the spectrum of the operator obtained by linearization of the right-hand side of (1.4) about the travelling wave $w(x)$,

$$Mu = a\Delta u + c\frac{\partial u}{\partial x_1} + F'(w(x), x')u, \ \left.\frac{\partial u}{\partial \nu}\right|_{\partial\Omega} = 0. \tag{1.5}$$

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