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ASYMPTOTIC STABILITY FOR THE DIFFERENTIAL SYSTEM $u'' + \sigma(t)|u|^\alpha|u'|^\beta u' + f(u) = 0$

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1. Introduction. We shall be concerned with the asymptotic stability of the rest state of the quasi-variational system

$$(\nabla\mathcal{L}(t,u,u'))' - \nabla_u\mathcal{L}(t,u,u') = Q(t,u,u'), \qquad J = [T,\infty), \tag{1.1}$$

where $u : J \to \mathbb{R}^N$ and $\mathcal{L}(t,u,p) = G(u,p) - F(t,u)$ and where

$$G \in C^1(\mathbb{R}^N \times \mathbb{R}^N;\mathbb{R}), \quad F \in C^1(J \times \mathbb{R}^N;\mathbb{R}), \quad Q \in C(J \times \mathbb{R}^N \times \mathbb{R}^N;\mathbb{R}^N).$$

We assume here that the nonlinear damping magnitude $|Q|$ is controlled from below by a function of the natural form $\sigma(t)\,\phi(u,p)$. The case when

$$\phi(u,p) > 0 \qquad \text{for} \quad p \neq 0 \tag{1.2}$$

has been treated extensively in the literature; see for example the work of Arstein and Infante, Burton, Levin and Nohel, Pucci and Serrin, Salvadori, Smith, Thurston and Wong.

In this paper we study the more general situation in which

$$\phi(u,p) > 0 \quad \text{for} \quad u \neq 0, \quad p \neq 0. \tag{1.3}$$

This is a relatively new case. When $G(p) = |p|^2/2$, and *only in the scalar case*, Yoshizawa ([17]) obtained some results when the function σ is bounded from zero and from above, while Ballieu and Peiffer in [2] studied the case $\phi(u,p) = \tau(u)|p|^\beta$ and $\sigma(t) \equiv 1$. More recently Pucci and Serrin ([13]) have extended and generalized these studies to quasi-variational *systems* of the general form (1.1). In their work the function σ is assumed to satisfy a mean integral condition of the type introduced by Hatvani ([4]). Although this is a rather mild assumption, since it allows σ to have the behavior $\liminf_{t\to\infty}\sigma(t) = 0$, it is not satisfied, however, by functions σ for which $\lim_{t\to\infty}\sigma(t) = 0$. Also, in the vectorial case their condition (V_1), namely

(V_1) *For all $U > 0$ and $p_0 > 0$ there is a nonnegative measurable function $h \notin L^1(J)$ such that*

$$|(Q(t,u,p),p)| \geq h(t) \quad \textit{for } t \in J,\ |u| \leq U,\ |p| \geq p_0,$$

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