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BLOW UP FOR $u_t - \triangle u = g(u)$ REVISITED

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1. Introduction. In this paper we are concerned with the relations between the existence of global, classical solutions of the evolution equation

$$\begin{cases} u_t - \triangle u = g(u) & \text{in} \quad (0,\infty)\times\Omega, \\ u = 0 & \text{on} \quad \partial\Omega, \\ u(0) = u_0 & \text{in} \quad \Omega, \end{cases} \tag{1}$$

and the existence of weak solutions of the stationary problem

$$\begin{cases} -\triangle u = g(u) & \text{in} \quad \Omega, \\ u = 0 & \text{on} \quad \partial\Omega. \end{cases} \tag{2}$$

Here, and throughout the paper, $\Omega \subset \mathbb{R}^N$ is a smooth, bounded domain and $g : [0,\infty) \to [0,\infty)$ is a C^1 convex, nondecreasing function. For *some* results, we will also assume that there exists $x_0 \geq 0$ such that $g(x_0) > 0$ and

$$\int_{x_0}^{\infty} \frac{ds}{g(s)} < \infty. \tag{3}$$

Solutions u of (1) and (2) are always assumed to be nonnegative. The initial condition u_0 is always assumed to be in $L^\infty(\Omega)$ and $u_0 \geq 0$, so that a classical solution of (1) exists on a maximal interval $(0, T_m)$.

By a weak solution of (2), we mean the following.

Definition 1. A weak solution of (2) is a function $u \in L^1(\Omega)$, $u \geq 0$, such that

$$g(u)\delta \in L^1(\Omega), \tag{4}$$

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