Advances in Differential Equations Volume 1, Number 5, September 1996, pp. 773 – 791

POSITIVE SOLUTIONS OF A SEMILINEAR ELLIPTIC EQUATION ON $\mathbb{R}^N$ WITH INDEFINITE NONLINEARITY

S. Cingolani* and J.L. Gámez†
Scuola Normale Superiore, Pisa, 56100 Italy

(Submitted by: A. Ambrosetti)

1. Introduction. In recent years several authors have investigated the "indefinite" semilinear problem

$$\begin{aligned} -\Delta u(x) &= \lambda u(x) + g(x) f(u(x)), \quad \text{for } x \in \Omega, \\ u &= 0, \quad \text{for } x \in \partial\Omega, \end{aligned} \tag{1.1}$$

where Ω is an open bounded regular domain, λ is a real parameter, g is a changing-sign function and f is superlinear nonlinearity (see [1, 6, 8, 9]).

The main purpose of this paper is to extend these existence and multiplicity results to $\mathbb{R}^N$.

Precisely, in this paper we look for positive solutions of the problem

$$\begin{aligned} -\Delta u(x) &= \lambda h(x) u(x) + g(x)|u(x)|^{p-2}u(x), \quad \text{for } x \in \mathbb{R}^N, \\ u &\in \mathcal{D}^{1,2}(\mathbb{R}^N), \end{aligned} \tag{1.2}$$

where λ is a real parameter, $N > 2$, $2 < p < 2^* = \frac{2N}{N-2}$.

To deal with (1.2), we shall assume throughout the paper that functions $g, h : \mathbb{R}^N \to \mathbb{R}$ satisfy the assumptions:

(H1) h is a continuous function, $h^+ \not\equiv 0$, $h \in L^\infty(\mathbb{R}^N) \cap L^{\frac{N}{2}}(\mathbb{R}^N)$;
(G1) g is a continuous function, $g \in L^\infty(\mathbb{R}^N) \cap L^q(\mathbb{R}^N)$, where $q = \frac{2N}{2N-pN+2p}$.

The role of (H1-G1) is to overcome the lack of compactness and to obtain some a priori bounds for λ. This is done in Section 2. In addition, if (H1) holds, it is well known that there exists the positive principal eigenvalue, $\lambda_1(h)$, of the corresponding eigenvalue problem

$$\begin{aligned} -\Delta u(x) &= \lambda h(x) u(x), \quad \text{for } x \in \mathbb{R}^N, \\ u &\in \mathcal{D}^{1,2}(\mathbb{R}^N). \end{aligned} \tag{1.3}$$

Received for publication December 1995.
*Supported by M.U.R.S.T., "Problemi Nonlineari..." and by CHRX-CT94-0494.
†Supported by ERBCHBGCT920011 Human Capital and Mobility Institutional fellowships. Permanent address: Departamento de Análisis Matemático, Universidad de Granada, Granada, Spain. The hospitality and support of Scuola Normale Superiore of Pisa are gratefully acknowledged.
AMS Subject Classifications: 35J20, 35J60.