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ON THE RATE OF DRYING OF GELATIN IN A PHOTOGRAPHIC FILM*

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1. Introduction. A simple photographic film consists of a mixture of gelatin and water. Many diffusion processes deal with species which occupy a negligible volume in comparison to the medium in which they diffuse (for example flow through a porous medium). This is not the case, however, with the diffusion of water in a gelatin emulsion. As water diffuses into (or out of) the gelatin emulsion, the emulsion can swell (or shrink) considerably. To avoid this problem a coordinate relative to the gelatin in its dry state is introduced. More precisely, if y is the Eulerian coordinate attached to the volume then the Lagrangian coordinate attached to the gelatin is given by

$$x = \int_0^y S(r,t)\,dr,$$

where $S(y,t)$ is the ratio of gelatin to total volume at the point y and time t. As the film dries the water diffuses more slowly and a *skin* of nearly pure gelatin forms on the surface. In order to evaporate the water must diffuse through this skin. This gives rise to some obvious questions. First, does the water evaporate completely or does the *skin* form quickly enough to stop the drying? Secondly, if the film does dry completely, at what rate does the water evaporate?

The following has been proposed as a model for this phenomena (see [5]):

$$\begin{cases} u_t = \frac{\partial}{\partial x}\big(G(u)\frac{\partial u}{\partial x}\big) & \text{in } \ (0,1)\times(0,\infty) \\ e^{-\frac{1}{u}}\frac{\partial u}{\partial x} + u = 0 & \text{at } \ x=1 \\ \frac{\partial u}{\partial x} = 0 & \text{for } \ x=0 \\ u(x,0) = u_0(x). \end{cases} \tag{1}$$

Here u represents the volume fraction of water, x is the Lagrangian coordinate attached to the gelatin, and

$$G(u) = \frac{e^{-\frac{1}{u}}}{(1+u)^2}.$$

Notice that as u tends to 0 the diffusion coefficient $G(u)$ tends to 0 as well, and thus as the water evaporates the diffusion slows down. For a more complete description of the

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