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ON THE WELL-POSEDNESS OF BENNEY'S INTERACTION EQUATION OF SHORT AND LONG WAVES

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1. Introduction. We study the initial-value problem for the system

$$\begin{cases} i\partial_t u + \partial_x^2 u = \alpha |u|^2 u + \beta n u, & t, x \in \mathbb{R}, \\ \partial_t n = \partial_x(|u|^2), \\ u(x,0) = u_0(x), \quad n(x,0) = n_0(x). \end{cases} \tag{1.1}$$

To describe the interaction between long and short water waves, Benney proposed two systems of dispersive equations ([1], [2]). One of the systems is

$$\begin{cases} i(\partial_t S + c_g \partial_x S) + \partial_x^2 S = \alpha |S|^2 S + \beta L S, & t, x \in \mathbb{R}, \\ \partial_t L + c_l \partial_x L = \gamma \partial_x(|S|^2), \end{cases} \tag{1.2}$$

where S and L denote the short wave envelope and the long wave profile, respectively; c_g, c_l, α, β and γ are real constants. When $\beta = 0$, this equation is a decoupled system and the first equation in (1.2) is the well-known cubic nonlinear Schrödinger equation which has been studied by many authors where both inverse scattering methods and nonlinear evolution equation techniques have been used. Further, N. Yajima-Oikawa used inverse scattering to obtain results for $\alpha = 0$ and $\beta > 0$ ([26]) (also see Ma [16] for the case $c_g = c_l$).

From the point of view of evolution equations, M. Tsutsumi-Hatano in [21] and [22] investigated the well-posedness, existence, uniqueness, persistence, and continuous dependence upon initial data of the Cauchy problem for Benney's equation (1.2) where the main concern is on the initial data S_0. They first established local well-posedness in fractional Sobolev spaces under the resonance condition $c_l = c_g$. When $\alpha = 0$ the initial-value problem is locally well-posed for $H^{1/2}(\mathbb{R})$; when $\alpha \in \mathbb{R}$ it is locally well-posed in higher spaces, $H^{j+(1/2)}$ where $j = 1, 2, 3, \ldots$. They improved these results to the nonresonance case, $c_g \neq c_l$, and also obtained global well-posedness when $\alpha = 0$ in the same spaces as above. In both of these results, the largest function space for the initial data S_0 is $H^{1/2}(\mathbb{R})$. This is the largest space in which the nonlinear term

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