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LARGE DEVIATIONS ESTIMATES FOR THE EXIT PROBABILITIES OF A DIFFUSION PROCESS THROUGH SOME VANISHING PARTS OF THE BOUNDARY

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Introduction. In the study of random perturbations of dynamical systems, the aim of the theory of Large Deviations is to give estimates on the events of "small probabilities"; we refer the reader to the works of Wentzell-Freidlin ([22]), Donsker-Varadhan ([13]) and Azencott ([1]) for an introduction and for the basic results of this theory.

In general, these events of "small probabilities" are the ones for which the behavior of the perturbed trajectories does not look like the behavior of the unperturbed trajectories. We are going in this article to investigate another possible source of "small probabilities" by estimating the exit probabilities of perturbed trajectories of a deterministic dynamical systems through "small" parts of the boundary of a domain in $\mathbb{R}^N$ which typically collapse to a point when the perturbation goes to zero; this is what we call "vanishing parts" of the boundary.

In order to be more specific, we consider Ω a smooth bounded domain of $\mathbb{R}^N$ and we denote by $(X_t^\varepsilon)_t$ the solution of the stochastic differential equation

$$dX_t^\varepsilon = b(X_t^\varepsilon)dt + \varepsilon\sigma(X_t^\varepsilon)dW_t\,, \qquad X_0^\varepsilon = x \in \Omega\,, \tag{1}$$

where $\varepsilon > 0$ and where b and σ are Lipschitz-continuous functions with values respectively in $\mathbb{R}^N$ and in the space of $N \times p$ matrices. Finally, $(W_t)_t$ denotes a standard p-dimensional Wiener process.

Then we consider $(\Gamma^\varepsilon)_\varepsilon$ a sequence of open subsets of $\partial\Omega$: the aim is to estimate the probability for a sample path of the solution of (1) to exit Ω through Γ^ε. To do so, we introduce the real-valued function u^ε defined on $\overline{\Omega}$ by

$$u^\varepsilon(x) = \mathbb{P}_x(X^\varepsilon_{\tau^\varepsilon_x} \in \Gamma^\varepsilon) = \mathbb{E}_x(\mathbb{1}_{\Gamma^\varepsilon}(X^\varepsilon_{\tau^\varepsilon_x}))\,,$$

where we denote respectively by $\mathbb{P}_x$ and $\mathbb{E}_x$ the conditional probability and the conditional expectation with respect to the event $\{X_0^\varepsilon = x\}$ and where τ_x^ε stands for the first exit time of $(X_t^\varepsilon)_t$ from Ω, i.e.,

$$\tau_x^\varepsilon = \inf\{t \geq 0,\ X_t^\varepsilon \notin \Omega\}\,,$$

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