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NONEXISTENCE OF POSITIVE SOLUTIONS OF QUASILINEAR EQUATIONS

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1. Introduction. In a series of papers Ni and Serrin ([12], [13], [14]) initiated the study of existence and nonexistence of positive radial solutions of quasilinear equations of the type

$$-\mathrm{div}\,(A(|\nabla u|)\nabla u) = f(u) \quad \text{in } \mathbb{R}^N. \tag{1.1}$$

Here $N \geq 3$, $A : \mathbb{R}^+ \to \mathbb{R}^+$ satisfies some structural assumptions and $f : \mathbb{R} \to \mathbb{R}$, roughly speaking, has a polynomial growth.

One of the typical results proved by Ni and Serrin reads as follows: Suppose that A is positive and bounded and

$$f(t) \geq c|t|^p \tag{1.2}$$

for $p > 1$, $c > 0$ and t near zero. If

$$1 < p \leq \frac{N}{N-2}, \tag{1.3}$$

then (1.1) has no positive radial solution such that ($r = |x|$),

$$\lim_{r\to\infty} u(r) = 0. \tag{1.4}$$

A solution of (1.1) satisfying (1.4) is called ground state.

The boundedness assumption on A covers many important cases. If $A = 1$, then (1.1) reduces to the canonical problem

$$-\Delta u = f(u) \quad \text{in } \mathbb{R}^N,$$

while the choice

$$A(t) = \frac{1}{\sqrt{1+t^2}} \tag{1.5}$$

corresponds to the mean curvature operator in nonparametric form.

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