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ABOUT THE C^∞-WELL-POSEDNESS OF FULLY NONLINEAR WEAKLY HYPERBOLIC EQUATIONS OF SECOND ORDER WITH SPATIAL DEGENERACY

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1. Introduction. The Cauchy problem for nonlinear strictly hyperbolic equations is C^∞ well-posed ([4]). Later the Cauchy problem was studied for not necessarily strictly hyperbolic equations; these are called weakly hyperbolic. We remind the reader in this context of [14], where the characteristics had to be of constant multiplicity or smooth. The case of characteristics of variable multiplicity was considered for linear hyperbolic equations in [23] and for systems of nonlinear hyperbolic equations in [13]. But in all these weakly hyperbolic cases the well-posedness of the Cauchy problem could be proved only in all Gevrey classes of order $s, 1 < s < d/(d-1)$, with respect to the spatial variables. Here d denotes the highest multiplicity of the characteristic roots of a suitable main symbol which are real in the weakly hyperbolic case. The critical exponent arises in a natural way. For that we have only to take into consideration the fact that the Cauchy problem for $u_t^{(m)} - u_x = 0$ is globally well-posed if and only if $1 \le s < m/(m-1)$ (for $m = 2$ see [8]). One has to assume so-called Levi conditions to include Gevrey classes of order $s > d/(d-1)$ or C^∞. To explain this kind of conditions we consider two model equations.

Let us be given the weakly hyperbolic equation with time degeneracy of finite order (t^l) $u_{tt} - t^{2l}u_{xx} + at^k u_x = 0$. It follows from [11, 12, 17] that the Cauchy problem is well-posed in all Gevrey classes and in C^∞ if and only if $k \ge l-1$. In opposition to this result the Cauchy problem for the weakly hyperbolic equation with spatial degeneracy of finite order (x^l) is well-posed in all Gevrey classes and in C^∞ if and only if $k \ge l$ ([11, 12, 19]). Both types of this kind of degeneracy were combined in [10, 19, 22, 24]. In the following we restrict ourselves to the case of spatial degeneracy. In [26, 27] a general concept of necessary and sufficient conditions for the C^∞ well-posedness was proposed even for higher-order weakly hyperbolic equations. Let us illustrate some conditions for the equation

$$u_{tt} + b(x,t)u_{xt} - a(x,t)u_{xx} + c(x,t)u_x + d(x,t)u_t + g(x,t)u = f(x,t). \qquad (1.1)$$

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