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LOCAL EXISTENCE AND UNIQUENESS OF WEAK SOLUTIONS FOR NONLINEAR PARABOLIC EQUATIONS WITH SUPERLINEAR GROWTH AND UNBOUNDED INITIAL DATA

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1. Introduction. A wide, and nowadays classical, literature has dealt with the Cauchy–Dirichlet problem for the superlinear heat equation:

$$\begin{cases} u_t - \Delta u = |u|^{p-1}u & \text{in } \Omega \times (0,T), \\ u = 0 & \text{on } \partial\Omega \times (0,T), \\ u(x,0) = u_0(x) & \text{in } \Omega, \end{cases} \tag{1.1}$$

where Ω is an open, bounded subset of $\mathbf{R}^N$, $p > 1$ and u_0 belongs to a Lebesgue space $L^q(\Omega)$ for some $q \geq 1$. Well–known examples show that a global (in time) solution of (1.1) can not in general be expected, so problem (1.1) needs to be formulated inside a maximal interval $(0, T_{\max})$, where $T_{\max}$ depends on u_0. Since the works by F. B. Weissler ([10] and [11]), several authors have investigated different features of this problem under the assumption that u_0 only belongs to $L^q(\Omega)$, with $q < +\infty$ (see [6], [9], [2], [4]), obtaining existence, uniqueness and continuous-dependence results for (1.1) always in the framework of linear semigroup theory and for the so–called mild, or integral, solutions, namely if u satisfies

$$u(t) = T_t u_0 + \int_0^t T_{t-s}|u(s)|^{p-1}u(s)\,ds\,, \tag{1.2}$$

where T_t denotes the heat semigroup. A condition is required in the link between q and p in order to have existence, that is either $q > \frac{N(p-1)}{2}$ and

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