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SHARP L^p–HODGE DECOMPOSITIONS FOR LIPSCHITZ DOMAINS IN $\mathbb{R}^2$

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1. Introduction and statement of main results

Let Ω be a bounded Lipschitz domain in $\mathbb{R}^2$, with outward unit normal $\vec{\nu} = \{\nu_1, \nu_2\}$ and unit tangent $\vec{\tau} = \{-\nu_2, \nu_1\}$ defined almost everywhere with respect to the surface measure $d\sigma$. Recall that a Lipschitz domain is a domain whose boundary is locally given by graphs of Lipschitz functions. For more details see, e.g., [7]. For $1 < p < \infty$, we denote by L^p the space of p-integrable functions (which will be defined either over Ω or $\partial\Omega$), and by $L^p_1(\partial\Omega)$ the space of functions in $L^p(\partial\Omega)$ with tangential gradients in $L^p(\partial\Omega)$. Also, $H^{r,p}(\Omega)$, $r \in \mathbb{R}$, denotes the usual scale of Sobolev spaces on Ω, while $H^{r,p}_0(\Omega)$ denotes the space of distributions in $H^{r,p}(\mathbb{R}^2)$ with support contained in $\bar{\Omega}$; cf. [20].

As usual, $\nabla := \{\partial_1, \partial_2\}$, and for a vector field $\vec{u} = \{u_1, u_2\}$ with locally integrable components in Ω we set

$$\operatorname{div} \vec{u} := \partial_1 u_1 + \partial_2 u_2, \quad \operatorname{rot} \vec{u} := \partial_1 u_2 - \partial_2 u_1, \quad \nabla^t u := \{\partial_2 u, -\partial_1 u\}, \quad (1.1)$$

where the derivatives are considered in the sense of distributions. The spaces of harmonic L^p vector fields with vanishing normal or tangential traces are

$$\mathcal{H}^p_{\tan}(\Omega, \mathbb{R}^2) := \{\vec{u} \in L^p(\Omega, \mathbb{R}^2) : \operatorname{div} \vec{u} = 0,\ \operatorname{rot} \vec{u} = 0 \text{ in } \Omega,\ \vec{\nu} \cdot \vec{u} = 0\}, \quad (1.2)$$

$$\mathcal{H}^p_{\text{nor}}(\Omega, \mathbb{R}^2) := \{\vec{u} \in L^p(\Omega, \mathbb{R}^2) : \operatorname{div} \vec{u} = 0,\ \operatorname{rot} \vec{u} = 0 \text{ in } \Omega,\ \vec{\tau} \cdot \vec{u} = 0\}. \quad (1.3)$$

Theorem 1.1. *Let Ω be a bounded Lipschitz domain in $\mathbb{R}^2$. There exists $\varepsilon = \varepsilon(\partial\Omega) \in (0, \frac{1}{4}]$ such that if $1/p_0 := \frac{3}{4} + \varepsilon$ and $1/p_0' := \frac{1}{4} - \varepsilon$, then for*

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