Advances in Differential Equations Volume 5 (1–3) January–March 2000, pp. 293–322

MAXIMAL REGULARITY IN L^p SPACES FOR AN ABSTRACT CAUCHY PROBLEM

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1. Introduction. In this paper we study the maximal L^p regularity for the Cauchy problem

$$\begin{cases} u'(t) = Au(t) + f(t), & t \in I \\ u(0) = 0 \end{cases} \tag{CP}$$

in a complex Banach space X, where A is a linear operator with domain and range contained in X, I is a bounded or unbounded interval with first extremum equal to 0 and $1 \leq p \leq \infty$.

Maximal L^p regularity means that for every $f \in L^p(I, X)$ (CP) has one and only one solution u belonging to $L^p(I, X)$ and such that u' and Au belongs to the same space and depends continuously on f.

The paper is arranged in the following way. In section 2 the notation is explained and some theorems that are employed in the sequel are recalled. In section 3 we collect some easy consequences of L^p regularity. In section 4 we prove that L^p regularity implies an estimate on the resolvent of the operator A; this estimate shows that A generates a (not necessarily strongly continuous) analytic semigroup. In Section 5 we prove that in some sense L^p regularity does not depend on the interval in which the Cauchy problem is considered. In Section 6 we prove two perturbation results. In Section 7 we give a proof of the (essentially already known) fact that L^p regularity does not depend on the exponent p (if $1 < p < \infty$). In Section 8 we give an example of an unbounded operator A in a "bad" space such that there is L^p regularity for (CP).

Received for publication July 1998.
AMS Subject Classifications: 34G10, 47D06.