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THE POROUS MEDIA EQUATION WITH NONCONSTANT COEFFICIENTS

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1. Introduction. In this paper we consider nonnegative solutions of the degenerate parabolic equation

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x} A(x,t) \frac{\partial u^m}{\partial x} \quad \text{in} \quad Q = \mathbb{R} \times \mathbb{R}^+ \ , \tag{1.1}$$

where $m > 1$ is a constant. In addition $A(x,t) \in L^\infty(Q)$ and

$$0 < \lambda \le A(x,t) \le \Lambda \tag{1.2}$$

for some positive constants λ and Λ which do not depend on x and t.

If $A(x,t) = 1$ for all $(x,t) \in Q$, then (1.1) is the classical porous media equation

$$u_t = (u^m)_{xx} \quad \text{in} \quad Q \ , \tag{1.3}$$

which has been studied extensively during the last three decades. Since $m > 1$, the diffusion coefficient mu^{m-1} vanishes at points where u vanishes, which leads to the existence of nonclassical solutions, and to the property of finite speed of propagation, i.e., if initially a solution has compact support, its support remains compact for all later times. Because of its simple structure, equation (1.3) has become a model equation for a large class of

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