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ON AXISYMMETRIC SOLUTIONS OF THE CONFORMAL SCALAR CURVATURE EQUATION ON S^n

Chiun-Chuan Chen
Department of Mathematics, National Taiwan University, Taipei, Taiwan

Chang-Shou Lin
Department of Mathematics, National Chung Cheng University
Chia Yi, Taiwan

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1. Introduction. Consider the sphere $S^n = \{(y_1, \dots, y_{n+1}) : y_1^2 + \dots + y_{n+1}^2 = 1\}$ with the standard metric $ds_0^2 = dy_1^2 + \dots + dy_{n+1}^2$. For $n \geq 3$, we consider the following problem of prescribing scalar curvature on S^n: given a smooth function K on S^n, we want to find a metric ds^2 conformal to ds_0^2 such that the given function K is the scalar curvature of the new metric ds^2. If we write $ds^2 = v^{\frac{4}{n-2}}\, ds_0^2$, then the problem of prescribing scalar curvature on S^n is equivalent to finding a positive smooth solution v of

$$\Delta_{g_0} v - \frac{n(n-2)}{4} v + \frac{n-2}{4(n-1)} K(y) v^{\frac{n+2}{n-2}} = 0 \quad \text{on } S^n\ , \tag{1.1}$$

where Δ_{g_0} is the Beltrami-Laplace operator associated with ds_0^2. By using $x \in \mathbf{R}^n$ to denote the coordinate of the stereographic projection of a point y on S^n and $u(x) = v(y)(\frac{2}{1+|x|^2})^{\frac{n-2}{2}}$, equation (1.1) reduces to

$$\begin{cases} \Delta u(x) + K(x) u^{\frac{n+2}{n-2}} = 0 & \text{in } \mathbf{R}^n, \\ u(x) = O(|x|^{2-n}) & \text{at } \infty \end{cases} \tag{1.2}$$

after an appropriate scaling. Recently, there have been many works denoted to this problem. We refer interested readers to [1], [2], [3], [9], [10], [11],...,

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