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CRITICAL EXPONENT PROBLEM IN $\mathbb{R}^2$-BORDER-LINE BETWEEN EXISTENCE AND NON-EXISTENCE OF POSITIVE SOLUTIONS FOR DIRICHLET PROBLEM

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1. Introduction. Let $\Omega \subset \mathbb{R}^2$ be a bounded domain with smooth boundary $\partial\Omega$. Let $B(R)$ denote the open disc of radius R centered at the origin in $\mathbb{R}^2$. Denote by $\lambda_1(\Omega)$ the first Dirichlet eigenvalue of $-\Delta$ on Ω. Consider the following nonlinear eigenvalue problem:

$$\begin{aligned} -\Delta u &= \lambda h(u) e^{u^2} && \text{in } \Omega\ , \\ u &> 0 && \text{in } \Omega\ , \\ u &= 0 && \text{on } \partial\Omega. \end{aligned} \tag{1.1}$$

Here the nonlinearity $h(s)$ is a lower-order perturbation of e^{s^2}. Note that such nonlinearities $f(s) = h(s)e^{s^2}$ are critical from the view point of the Trudinger imbedding. It is easy to show that a necessary condition for (1.1) to admit a solution on any domain Ω is that $\lambda \in (0, \frac{\lambda_1(\Omega)}{f'(0)})$. Regarding the problem (1.1), Brezis [6] has raised the following question:

Q: "Where is the border-line between existence and non-existence in (1.1) ?"

Our answer to this question will be

A: (1.1) has a solution $\forall \lambda \in (0, \frac{\lambda_1(\Omega)}{f'(0)}) \Leftrightarrow \overline{\lim}_{s\to\infty} h(s)s = \infty$.

Here we write $\Leftrightarrow$ since the non-existence part is proved in this paper in the radial situation for λ close to 0 and when the perturbation h is convex

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