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NONLINEAR ELLIPTIC EQUATIONS IN $\mathbb{R}^N$ WITH "ABSORBING" ZERO ORDER TERMS

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0. Introduction. In [1] the semilinear elliptic problem

$$\begin{cases} -\Delta u + |u|^{s-1}u = f & \text{in } \mathcal{D}'(\mathbb{R}^N)\,, \\ \qquad u \in L^s_{loc}(\mathbb{R}^N)\,, \end{cases} \tag{0.1}$$

having merely *locally* integrable datum $f \in L^1_{loc}(\mathbb{R}^N)$, and equipped with *no prescribed behaviour at infinity* of u, has been shown to have a unique solution u whenever the exponent s satisfies $s > 1$.

This result has been subsequently generalized in [10], where the power-like nonlinearity $t \mapsto |t|^{s-1}t$ was replaced by a general function $g(t)$ satisfying the following hypotheses:

$$g : \mathbb{R} \mapsto \mathbb{R} \ \text{ is continuous, odd, increasing, } \ g(0) = 0 \tag{$G1$}$$

$$g \ \text{ is convex in } \ [0, +\infty) \tag{$G2$}'$$

$$\int^{+\infty} \frac{dt}{[tg(t)]^{\frac{1}{2}}} < +\infty\,. \tag{$G3$}'$$

Under these assumptions the problem

$$\begin{cases} -\Delta u + g(u) = f & \text{in } \mathcal{D}'(\mathbb{R}^N)\,, \\ \quad u\,, g(u) \in L^1_{loc}(\mathbb{R}^N) \end{cases} \tag{0.2}$$

has been shown to have a unique solution.

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