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SCATTERING SOLUTIONS FOR PLANAR SINGULAR HAMILTONIAN SYSTEMS VIA MINIMIZATION

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0. Introduction. In a recent paper [6] the authors considered the existence of unbounded solutions for second-order, singular Hamiltonian systems of the form

$$\ddot{q} + \nabla V(q) = 0, \tag{0.1}$$

where V has a singularity at the origin behaving like $-1/|q|^\alpha$, with $\alpha > 2$, the strong force case. Since the system is autonomous, the energy is conserved along the solutions of (0.1); that is, there is a value H such that

$$\frac{1}{2}|\dot{q}|^2 + V(q) = H. \tag{0.2}$$

When $H > 0$, it is shown in [6] that unbounded solutions exist. More precisely it is proved that given any $H > 0$, $\hat{\theta}_1, \hat{\theta}_2 \in S^{N-1}$, $\hat{\theta}_1 \neq -\hat{\theta}_2$, there exists a solution $q(t)$ of (0.1)–(0.2) also satisfying

$$|q(t)| \to +\infty \quad \text{if} \quad t \to \pm\infty, \tag{0.3}$$

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