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ON THE EXISTENCE OF SOLUTIONS OF HAMILTONIAN ELLIPTIC SYSTEMS IN $\mathbb{R}^N$

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(Submitted by: James Serrin)

1. Introduction and main results. In this paper we study the following elliptic system of Hamiltonian type

$$\begin{cases} -\Delta u + b(x)u &= g(x,v) \\ -\Delta v + b(x)v &= f(x,u), \qquad x \in \mathbb{R}^N, \end{cases} \tag{1}$$

where $N \geq 3$ and $f, g : \mathbb{R}^N \times \mathbb{R} \to \mathbb{R}$, $b : \mathbb{R}^N \to \mathbb{R}$ are given continuous functions. We suppose that f and g are subcritical (see below).

Systems of this kind in a bounded domain, even with more general nonlinearities, were studied in a number of papers (see [5], [7], [10], [13] and the references in these works; supercritical systems were treated in [18]—see the remarks at the end of this section). The case of the whole space was first considered in a recent paper by de Figueiredo and Yang [11]. They studied the system

$$\begin{cases} -\Delta u + u &= g(x,v) \\ -\Delta v + v &= f(x,u), \qquad x \in \mathbb{R}^N, \end{cases} \tag{2}$$

proving under some conditions the existence of a strong radial solution (Theorem 5.1 in [11]). They also showed that strong solutions of (2) decay at infinity, together with their derivatives (Theorem 2.1 in [11]; see below for a definition of a strong solution).

Studying Theorems 2.1 and 5.1 in [11], it appeared to us that they require a number of hypotheses which seem too strong, when compared to the well-known conditions under which existence and regularity of solutions of a

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