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BLOWUP BEHAVIOR OF SOLUTIONS TO THE RESCALED JÄGER-LUCKHAUS SYSTEM

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1. Introduction

In this paper, we consider the blowup behavior of solutions to the rescaled Jäger-Luckhaus system.

In [9], Jäger and Luckhaus introduced the following parabolic-elliptic system.

$$\begin{array}{lll} u_t = \nabla \cdot (\nabla u - u \nabla v) & \text{in} & \Omega \times (0,T) \\ 0 = \Delta v + u - \lambda/|\Omega| & \text{in} & \Omega \times (0,T) \\ \partial u/\partial \nu = \partial v/\partial \nu = 0 & \text{on} & \partial\Omega \times (0,T) \\ \int_\Omega v dx = 0 & \text{in} & [0,T) \\ u|_{t=0} = u_0 & \text{in} & \Omega. \end{array} \tag{1.1}$$

Here, $\Omega \subset \mathbf{R}^2$ denotes a bounded domain with smooth boundary $\partial\Omega$, ν is the outer normal unit vector, and $|\Omega|$ is the area of Ω. The initial value u_0 is smooth, nonnegative, and nontrivial. Let $\lambda = \|u_0\|_{L^1(\Omega)} > 0$. Here and henceforth, $\|\cdot\|_{L^p(\Omega)}$ denotes the standard $L^p(\Omega)$ norm for $1 \leqq p \leqq \infty$.

We refer to (1.1) as the Jäger-Luckhaus system. The Jäger-Luckhaus system is a simplified system compared to the one introduced by Keller and Segel [10] or Nanjundiah [14]. Those systems describe the chemotactic feature of some organisms (cellar slime molds) sensitive to the gradient of a chemical substance secreted by themselves. We refer to the one introduced by Nanjundiah [14] as the Keller-Segel model.

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