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EXISTENCE AND MULTIPLICITY RESULTS FOR A CLASS OF SCHRÖDINGER EQUATIONS WITH INDEFINITE NONLINEARITIES

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Introduction

In this paper we are concerned with existence and multiplicity results for a class of nonlinear Schrodinger equation of the form

$$(P) \qquad -\Delta u + V(x)u = a(x)g(u), \qquad x \in \mathbb{R}^N,$$

where $V(x) \in C(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$ and $a(x) \in C(\mathbb{R}^N)$. The nonlinear term $g(u)$ will always be assumed to have superlinear behavior at zero, a power-like growth at infinity, and the sign condition $g(u)u \geq 0$, for all $u \in \mathbb{R}$. Precise hypotheses will be stated in Section 1.

Regarding the weight function $a(x)$, we consider the setup introduced in our earlier work [3]. In fact we assume that $a(x)$ changes sign in $\mathbb{R}^N$, thus the indefinite character of the nonlinearity, and

$$(I_1) \qquad \lim_{|x|\to\infty} a(x) = a_\infty.$$

We note that in case $a(x)$ is *positive at infinity*, that is, $\lim_{|x|\to\infty} a(x) = a_\infty > 0$, it can be seen that Pohozaev-type identities provide *nonexistence* results under rather mild assumptions. Therefore we further assume

$$(I_2) \qquad \lim_{|x|\to\infty} a(x) = a_\infty < 0.$$

In case the Schrodinger operator $L = -\Delta + V(x) : H^2(\mathbb{R}^N) \to L^2(\mathbb{R}^N)$ is nonnegative, that is, $\sigma(L) \cap (-\infty, 0) = \emptyset$, there is a number of existence and multiplicity results for (P) under various assumptions on the weight function $a(x)$ and the nonlinearity $g(u)$. Of particular interest, however, is when $\sigma(L) \cap (-\infty, 0) \neq \emptyset$, so that the associated quadratic form $Q(u) = \frac{1}{2}(Lu, u)$

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