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ON THE MINIMIZERS OF A GINZBURG-LANDAU-TYPE ENERGY WHEN THE BOUNDARY CONDITION HAS ZEROS

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1. Introduction

Let Ω be a smooth, bounded, and simply connected domain in $\mathbb{R}^2$, and let g be a smooth boundary condition which takes its values in $\mathbb{R}^2$. We are interested in the asymptotic behavior, as ε goes to 0, of the minimizers $\{u_\varepsilon\}$ of the Ginzburg-Landau-type energy

$$E_\varepsilon(u) = \int_\Omega |\nabla u|^2 + \frac{1}{\varepsilon^2}\int_\Omega (1-|u|^2)^2$$

over the set $H^1_g(\Omega,\mathbb{R}^2) = \{u \in H^1(\Omega,\mathbb{R}^2);\ u = g \text{ on } \partial\Omega\}$. When g is S^1-valued the problem was resolved in the work of Bethuel, Brezis, and Hélein [4, 5] (see also Struwe [17] and Bethuel-Rivière [6]). There it was shown that if the degree of g is $d > 0$ (the case $d = 0$ is simpler; see [4]), then there exist d distinct points $a_1, \ldots, a_d \in \Omega$ such that, for a subsequence $\varepsilon_n \to 0$, we have $u_{\varepsilon_n} \to u_*$ in $C^{1,\alpha}_{\mathrm{loc}}(\overline{\Omega} \setminus \{a_1, \ldots, a_d\})$, where $u_* \in C^\infty(\overline{\Omega} \setminus \{a_1, \ldots, a_d\}, S^1)$ is a singular harmonic map which has degree 1 around each a_j (it is called the *canonical harmonic map* associated to the points $a_1, \ldots, a_d$, the configuration of degrees $(1, \ldots, 1)$, and the boundary condition g; see [5]). Moreover, the following estimate for the energy holds:

$$E_\varepsilon(u_\varepsilon) = 2\pi d|\log\varepsilon| + O(1) \text{ as } \varepsilon \to 0.$$

In our previous work [2], we studied the case of a boundary condition g which is not necessarily S^1-valued, but which is not allowed to take the value 0. Under this assumption (i.e., $g \in C^2(\partial\Omega, \mathbb{R}^2 \setminus \{0\})$), when d, the degree

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