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HELICOIDAL TRAJECTORIES OF A CHARGE IN A NONCONSTANT MAGNETIC FIELD

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1. Introduction

In this note we investigate the existence of helicoidal trajectories for a charged particle in a magnetic field. More precisely, denoting by $p = p(t)$ the position in $\mathbb{R}^3$ at the time t of the particle, we say that it moves along a helicoidal trajectory if there exists a versor n in $\mathbb{R}^3$ such that the component of $p(t)$ in the direction of n describes a uniform right motion, whereas the projection $p_\perp(t)$ of $p(t)$ on a plane orthogonal to n is periodic. In particular, if the closed curve supported by $p_\perp$ is simple; i.e., it has no self-intersections, the helicoidal trajectory $p(t)$ will be called simple.

From classical physics, in the presence of an external magnetic field B, the motion of a particle of mass m and charge e is driven by the Lorentz force, namely $p(t)$ is a solution of

$$m\ddot{p} = e\dot{p} \wedge B. \tag{1.1}$$

When B is a uniform, constant field, namely $B = b_0 n$ for some versor n and nonzero constant b_0, one can explicitly solve (1.1) and deduce that the particle admits helicoidal trajectories $p(t)$ which are coaxial with the magnetic field B. In particular the projection $p_\perp(t)$ of $p(t)$ on a plane orthogonal to B moves on a circle of radius r with constant angular speed ν. The values of ν and r are given respectively by

$$\nu = \frac{|eb_0|}{m}, \quad r = \frac{|\dot{p}_\perp(0)|}{\nu}. \tag{1.2}$$

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