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THE WORK OF WŁADYSŁAW NARKIEWICZ IN NUMBER THEORY AND RELATED AREAS

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All but four Narkiewicz's research papers and monographs concerning number theory deal with one of the following five topics

1. polynomial mappings,
2. arithmetical functions,
3. additive problems,
4. factorization in algebraic number fields,
5. Artin's conjecture in algebraic number fields and related topics.

We shall consider these topics successively, then deal with the four papers out of the above classification and finally consider the four big books by the author.

1. Here belong papers [3], [5], [10], [12], [13], [68], [70]–[73], [77], [80]–[82], [85], [86], [90], [93], [94] and the book [78]. For a field k a polynomial mapping $F : k^n \to k^n$ defined by

$$[x_1, \dots, x_n] \mapsto [f_1(x_1, \dots, x_n), \dots, f_n(x_1, \dots, x_n)]$$

is called admissible, if none of the polynomials $f_1, \dots, f_n$ is linear and their leading forms do not have any non-trivial common zero in the algebraic closure of k. A field k is said to have the property (SP), if for every n and every admissible polynomial mapping $F : k^n \to k^n$ the conditions $X \subset k^n$, $F(X) = X$ imply the finiteness of X. If this implication holds in the case $n = 1$, then k has property (P). Further, k has property (R), if the conditions $X \subset k$, X infinite, $f \in k(T)$ and $f(X) = X$ imply

$$f(T) = \frac{\alpha + \beta\, T}{\gamma + \delta T}; \quad \alpha, \beta, \gamma, \delta \in k.$$

Finally, k has property (K), if the following is true.

Let $\Phi : k^n \to k^n$ be an admissible polynomial mapping and let $\Psi : k^n \to k^n$ be another polynomial mapping. Denote by d the minimum of degrees of polynomials defining Φ and D the maximum of degrees of polynomials defining Ψ. If $d > D$, A