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DISCRETE SERIES REPRESENTATIONS OF p-ADIC GROUPS ASSOCIATED TO SYMMETRIC SPACES

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1. Introduction

The purpose of this paper is study the natural symmetric space analogues of various notions related to discrete series representations of a p-adic group such as Schur's orthogonality relations and formal degrees.

We study representations of the group $G = \mathbf{G}(F)$ of F-rational points of a connected, reductive F-group $\mathbf{G}$, where F is a finite extension of a field $\mathbb{Q}_p$ of p-adic numbers for some odd prime p.

The representations of interest are associated to a symmetric space $H\backslash G$, where $H = \mathbf{H}(F)$ and $\mathbf{H}$ is the group of fixed points of some F-automorphism θ of $\mathbf{G}$ of order two.

To be more precise, we are interested in irreducible admissible complex representations (π, V) of G that are H-*distinguished* in the sense that there exists a nonzero linear form $\tilde{\lambda} : V \to \mathbb{C}$ that is H-fixed or, in other words,

$$\langle \pi(h)v, \tilde{\lambda}\rangle = \langle v, \tilde{\lambda}\rangle,$$

for all $h \in H$ and $v \in V$. From now on, assume that such a representation (π, V) has been fixed. Note that H-distinction[1] implies that the restriction of the central quasi-character of π to Z_H is trivial.

The latter linear forms, together with 0, comprise the space $\mathrm{Hom}_H(\pi, 1)$ and Frobenius Reciprocity maps this space isomorphically onto the space

$$\mathrm{Hom}_G(\pi, C^\infty(H\backslash G)),$$

where $C^\infty(H\backslash G)$ is the space of smooth complex-valued functions on $H\backslash G$ viewed as a G-module with respect to right translations by G. So π is H-distinguished precisely when it has a G-invariant embedding in $C^\infty(H\backslash G)$. In this sense, the H-distinguished representations are precisely the representations that contribute to the harmonic analysis on $H\backslash G$.

For convenience, we will make some simplifying assumptions that are generally satisfied in applications. Let $(\tilde{\pi}, \widetilde{V})$ be the contragredient of (π, V). We assume that $\mathrm{Hom}_H(\tilde{\pi}, 1)$, in addition to $\mathrm{Hom}_H(\pi, 1)$, is nonzero and, furthermore, we assume that both of the latter spaces are finite-dimensional.

Let $\mathbf{Z}$ be the center of $\mathbf{G}$ and let $\mathbf{Z_H} = \mathbf{Z}\cap\mathbf{H}$ and let $Z = \mathbf{Z}(F)$ and $Z_H = \mathbf{Z_H}(F)$. We fix a Haar measure on H/Z_H for use in our integrations over the latter quotient.

[1] At the suggestion of Hervé Jacquet, we refer to the property of being H-distinguished as "H-distinction," rather than "H-distinguishedness."