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THE INTRINSIC FLAT DISTANCE BETWEEN RIEMANNIAN MANIFOLDS AND OTHER INTEGRAL CURRENT SPACES

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Abstract

Inspired by the Gromov-Hausdorff distance, we define a new notion called the intrinsic flat distance between oriented m dimensional Riemannian manifolds with boundary by isometrically embedding the manifolds into a common metric space, measuring the flat distance between them and taking an infimum over all isometric embeddings and all common metric spaces. This is made rigorous by applying Ambrosio-Kirchheim's extension of Federer-Fleming's notion of integral currents to arbitrary metric spaces.

We prove the intrinsic flat distance between two compact oriented Riemannian manifolds is zero iff they have an orientation preserving isometry between them. Using the theory of Ambrosio-Kirchheim, we study converging sequences of manifolds and their limits, which are in a class of metric spaces that we call integral current spaces. We describe the properties of such spaces including the fact that they are countably \mathcal{H}^m rectifiable spaces and present numerous examples.

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