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CONSTRUCTIONS OF SMALL SYMPLECTIC 4-MANIFOLDS USING LUTTINGER SURGERY

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Abstract

Luttinger surgery is used to produce minimal symplectic 4-manifolds with small Euler characteristics. We construct a minimal symplectic 4-manifold which is homeomorphic but not diffeomorphic to $\mathbb{CP}^2\#3\overline{\mathbb{CP}}^2$, and which contains a genus two symplectic surface with trivial normal bundle and simply-connected complement. We also construct a minimal symplectic 4-manifold which is homeomorphic but not diffeomorphic to $3\mathbb{CP}^2\#5\overline{\mathbb{CP}}^2$, and which contains two disjoint essential Lagrangian tori such that the complement of the union of the tori is simply-connected.

These examples are used to construct minimal symplectic manifolds with Euler characteristic 6 and fundamental group $\mathbb{Z}$, $\mathbb{Z}^3$, or $\mathbb{Z}/p \oplus \mathbb{Z}/q \oplus \mathbb{Z}/r$ for integers p, q, r. Given a group G presented with g generators and r relations, a symplectic 4-manifold with fundamental group G and Euler characteristic $10 + 6(g + r)$ is constructed.

1. Introduction

In this article we construct a number of small (as measured by the Euler characteristic e) simply connected and non-simply connected smooth 4-manifolds which admit symplectic structures. Specifically, we construct examples of:

- A minimal symplectic manifold X homeomorphic but not diffeomorphic to $\mathbb{CP}^2\#3\overline{\mathbb{CP}}^2$ containing symplectic genus 2 surface with simply connected complement and trivial normal bundle, and a disjoint nullhomologous Lagrangian torus (Theorem 13).
- A minimal symplectic manifold B homeomorphic but not diffeomorphic to $3\mathbb{CP}^2\#5\overline{\mathbb{CP}}^2$ containing a disjoint pair of symplectic tori with simply connected complement and trivial normal bundle (Theorem 18). This provides a smaller substitute for the elliptic

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