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THE DEFORMATION OF LAGRANGIAN MINIMAL SURFACES IN KÄHLER-EINSTEIN SURFACES

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A Kähler manifold can be viewed both as a symplectic manifold and a Riemannian manifold. These two structures are related by the Kähler form. One can study the Lagrangian minimal submanifolds which are Lagrangian with respect to the symplectic structure and are minimal with respect to the Riemannian structure. Lagrangian minimal submanifolds have many nice properties and have been studied by several authors (see [3], [5], [13], [16], [17], [27], [30], [33], [34] etc.). There are obstructions to the existence of the Lagrangian minimal submanifolds in a general Kähler manifold [3]. These obstructions do not occur in a Kähler-Einstein manifold. But even in this case, the general existence is still unknown. Most of the discussions of the paper are on compact manifolds without boundary. We assume this from now on unless other conditions are indicated. The main result of this paper is the following:

Theorem 4. *Assume that (N, g_0) is a Kähler-Einstein surface with negative first Chern class. Let $[A]$ be a class in the second homology group $H_2(N, Z)$, which can be represented by a finite union of branched Lagrangian minimal surfaces with respect to the metric g_0. Then with respect to any other metric in the connected component of g_0 in the moduli space of Kähler-Einstein metrics, the class $[A]$ can also be represented by a finite union of branched Lagrangian minimal surfaces.*

Note that the complex structure on N is allowed to change accordingly. An immersed Lagrangian minimal submanifold in a Kähler manifold with negative Ricci curvature is strictly stable ([4], [20], [22]). Thus

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