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PRECISELY INVARIANT COLLARS AND THE VOLUME OF HYPERBOLIC 3–FOLDS

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Abstract

We give a lower bound for the volume of any hyperbolic 3–orbifold which admits an embedded tubular neighborhood of a closed geodesic. This bound depends *only* on the radius of this neighbourhood and not on the length of the geodesic. In the Kleinian group uniformizing such a hyperbolic 3–fold, this yields a lower bound on the co–volume purely in terms of the radius of a precisely invariant collar of a loxodromic axis. As an application of these results we obtain substantial improvements in the known volume bounds of hyperbolic 3–manifolds and certain orbifolds. This paper forms part of our program to identify the minimal volume hyperbolic 3–fold.

1. Introduction

Hyperbolic 3–space is the set

$$\mathbb{H}^3 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_3 > 0\}$$

endowed with the complete Riemannian metric $ds = |dx|/x_3$ of constant curvature equal to -1. We denote the hyperbolic distance in $\mathbb{H}^3$ by $\rho(\cdot, \cdot)$. A *Kleinian group* G is a discrete nonelementary subgroup of $Isom^+(\mathbb{H}^3)$, where $Isom^+(\mathbb{H}^3)$ is the group of orientation preserving isometries. In this setting *nonelementary* means that the group G is not virtually abelian. A *hyperbolic 3–orbifold* $\mathcal{Q}$ is the orbit space of a Kleinian group. Thus

$$\mathcal{Q} = \mathbb{H}^3/G. \tag{1.1}$$

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