

PU(2) MONOPOLES. I: REGULARITY, UHLENBECK COMPACTNESS, AND TRANSVERSALITY

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1. Introduction

At seminars at Harvard and MIT, during October 1994, Edward Witten introduced the $U(1)$ monopole equations and the Seiberg-Witten invariants to smooth four-manifold topology and conjectured their relationship with Donaldson invariants on the basis of new developments in quantum field theory [19], [98]. The conjecture, recently extended in [60], has been verified for all four-manifolds whose Donaldson and Seiberg-Witten invariants have been independently computed. Within two months of Witten's announcement, a program was outlined by V. Pidstrigach and A. Tyurin and others, which should lead to a mathematical proof of the relationship between these two invariants [68], [71], [74]. This approach is unrelated to the quantum field-theoretic arguments of [60], [98] and uses a moduli space of $PU(2)$ monopoles to construct a cobordism between links of Seiberg-Witten moduli spaces of $U(1)$ monopoles and the Donaldson moduli space of anti-self-dual connections, which appear as singularities in this larger stratified moduli space.

It was soon recognized, however, that despite the appeal and elegance of the $PU(2)$ monopole program, its implementation involves substantial technical difficulties due to the contributions of moduli spaces of $U(1)$ monopoles in the lower levels of the Uhlenbeck compactification of the moduli space of $PU(2)$ monopoles. Many of these difficulties had

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